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DEPARTMENT OF DEFENSE

LOGISTICS RESEARCH CONFERENCE

VOLUME II - 7

Individual Professional Papers
Pertaining to Panel 7 Discussion
On COST EFFECTIVENESS MODELS
AND EVALUATION

FOREWORD

The *individual professional papers* contained in this volume were prepared for presentation to the Panel on Cost Effectiveness Models and Evaluation (Panel 7) at the Department of Defense Logistics Research Conference held at the Airlie Conference Center, Warrenton, Virginia, 26-28 May 1965. They are published for information and background in relation to Panel 7 deliberations.

Publication of these individual professional papers by the Department of Defense does not necessarily signify that they represent a consensus of the Panel on Cost Effectiveness Models and Evaluation or the views of the Office of the Secretary of Defense.

Nathan Brodsky
NATHAN BRODSKY
Conference Chairman

CONTENTS

COST/EFFECTIVENESS ANALYSIS WITHIN THE ARMY

Oscar M. Wells

The primary purpose for performing cost/effectiveness studies, wherever they may be performed, is to provide a sounder, clearer, and more quantitative basis for decision making ... not to replace decision making (1), (8). Of course, the nature of the decision called for is determined largely by the organizational level for which the C/E study is performed. Normally the decision called for is a choice between candidate military systems for acquisition. (It is conventional to call these candidates "alternative weapons systems" although there may be several options and although these may not involve a weapon as such.) However, some studies which have been given the name C/E have only dealt with the question of economic utilization of operational systems (9).

Perhaps some would quarrel with the use of the term C/E study as applied to some of the following special studies. Their common element is that they involve an examination of the value obtained through the investment of a set of resources. We submit that C/E studies are performed within the Army for purposes as small as helping to decide between design trade-offs in a weapon system, and as comprehensive as helping to decide which of several alternative force structures to acquire and support.

The achievement of these purposes is best served when these studies focus early on the hard choices to be made between competing alternatives and possibly as a biproduct of such analysis suggest other alternatives (2). The function of the analyst is to present, in as concise a form as is possible, the costs of each of the alternatives and the expected military values to be achieved through their acquisition. It is precisely the expected value of acquisition that should rationally be compared with cost rather than the potential functional capability. Normally one pays increased marginal costs for increments in functional capability. However, if the tactical significance of these performance increments is great, i.e., if the increase in utility is great, greater investment cost may be warranted. This point has not always been clearly understood.

Although a great deal of attention has been given to C/E studies of strategic retaliatory and strategic airlift and sealift systems, a great deal of concern recently has been with tactical ground force systems such as the main battle tank (4) and (5) and the principal small arm weapons system (3) as well as with Army tactical air support of ground forces (7). As mentioned by Niskanen (8) analysis of such systems has been more difficult in many respects than the analysis of large strategic systems and consequently analytical support has lagged.

Efforts to come to grips with some of the conceptual problems--to be mentioned later--have been made, (5) and (6), but not with complete success; and much needs to be done to make analysis more valid and dependable.

As an example of the types of Army C/E studies recently reported in a special bibliography on cost/effectiveness by the Defense Logistics Studies Information Exchange, we can make the following observations. We divided the 20 titles, felt to be pertinent to C/E, into three categories: (1) materiel or case studies, (2) studies dealing with methodological and philosophical problems, and (3) studies performed to serve as guidance in performing C/E studies or in serving as an orientation to C/E. Seven (7) titles dealt with case studies; eight (8) primarily with methodology; and five (5) served as educational vehicles. Of the eight (8) methodological papers, six (6) dealt exclusively with costing.

Now I would like to turn to the consideration of three classes of problems: first, those inherently concerned with the evaluation of effectiveness; second, those concerned with the evaluation of costs; and thirdly those problems which are common to both cost and effectiveness evaluations. I shall attempt to indicate wherein the treatment of these problems has fallen short in our experience.

It is clear that the effectiveness of a weapons system can only be evaluated with certainty after it has been retired from operational use. Even then these would only be subjective and mainly qualitative answers to the question of the weapon's effectiveness. The fact is that there is no well defined quantitative expression for effectiveness. What seems to be indicated from the context in which that term is used by Hitch (2) and other authorities is that effectiveness is the likely value that the system is to provide or contribute to the total defense effort. This military value stems from the ability of the system to successfully perform the set of military tasks or missions assigned it, the more important missions being given emphasis. The performance of these military tasks is obviously related to measures of function. These measures, or more exactly, indices of functional capability such as speed, range, payload, etc. are often themselves called measures of--or components of effectiveness. Although the application of this term in the latter manner is questionable, it is in widespread use and will be so used here.

Problems arise, however, when the analyst attempts to relate such elementary measures to less well defined measures such as vulnerability, flexibility, maneuverability, etc. Problems also arise when one seeks to relate all the components of effectiveness and to evaluate their relative importance in the accomplishment of the set of military missions.

To summarize the problems just mentioned:

(1) to define the set of missions

(2) to rank these missions as to importance

(3) to identify and quantify the components of effectiveness

(4) to relate these to each other and to the level of mission accomplishment

The manner in which these problems is approached varies across Army studies. Missions most often are defined by direct appeal to the military user, but sometimes on the basis of pertinent historical evidence. In the former case there is not always accord from authority to authority and one has to negotiate or appeal to the decision maker. In the latter case, there is a great risk that the set of future missions for a system will differ from the past. In any case, it is much easier to define missions than to assign them relative importance. Since the result of this act is so dependent upon the value judgment and experience of the individual, one only has recourse to an opinion poll of authoritiative users. Often this latter step is simply ignored, all missions being treated as equally important, in one case (7), or only the principal mission considered, in the other case (3).

The problem of identifying and quantifying the components of effectiveness has been handled primarily by an appeal to the judgment of the analyst who is familiar with system operation and the set of missions. His skill and insight are indispensable here:

If all of the forementioned problems have been attacked competently, the very difficult problem remains of relating the components of effectiveness to each other and to the level of mission accomplishment. Several ways of accomplishing this can be identified:

(1) the user opinion poll

(2) a combat model or submodel

(3) a combat simulation or a war game

(4) a field exercise

Each method has its advantages and disadvantages. From the point-of view of being quickly responsive, a combat model had distinct advantages. It also provides quantitative outputs that can be verified by well designed field experiments. The latter aspect is important and is not provided by method (1).

The use of any of the four methods brings up explicitly or implicitly the question of how we stand competitively in the accomplishment of the missions for which these systems were designed. Because of the difficulty in obtaining adquate intelligence, this is an area of considerable uncertainty.

There is a continuing problem in most C/E studies not merely to acquire intelligence concerning the enemy's current materiel resources but to predict what they will be when our prototypes are operational. Unfortunately many Army C/E studies have not adequately dealt with this problem. The way in which various possible future threats may influence a decision to acquire a military system is not always clear-cut. For example, if a reasonable contingency is that the nature of the threat will change so as to remove a substantial number of the missions for which the system (s) were designed, the decision maker may wish to defer making a decision pending introduction of other alternatives or he may decide to go with one of the current alternatives, taking the risk that system life will be drastically shortened. In the area of strategic air defense systems such a decision-making environment was brought about by the enemy's shift from manned bombers to ballistic missiles. Therefore, it is important to explicitly ask the question of whether the mission(s) for which the system was designed are likely to persist throughout a reasonable, useful life of the system.

Turning to the area of costing, we encounter equally vexing problems. It is generally conceded to be desirable to approach cost/effectiveness by pivoting on cost, i.e., by scaling each of the alternatives to the same total expected cost. The principal advantage of doing this is to avoid the necessity of defining military worth in absolute terms. Also elements which are difficult to quantify may be left in qualitative terms. It is easier to merely rank the effectiveness of alternatives than to evaluate them.

There are two problems relative to implementing a cost pivot which are serious. The first is concerned with the question of scaling unequal alternatives to the same expected cost. And, the second is concerning with explicitly displaying the variability associated with making estimates of future costs. Obviously, alternatives are <u>really</u> constant as to cost only when they have nearly the same distribution of expected costs. In the event these distributions are not the same, the analyst can alter numbers of units, numbers of supporting personnel, size of mobilization base, configurational aspects of hardware, etc., within limits, so as to make expected costs constant over alternatives. After doing this, he may still see considerable differences in cost variability among alternatives. The analyst must decide whether the costs can be meaningfully scaled and whether differences in cost variability invalidate this procedure. One weakness of Army C/E studies to date has been the failure to deal sensibly with the latter problems.

Among analysts within the Army, there has been some confusion as to the treatment of inherited assets. For example, that part of the cost of training for a system in use that is applicable to its replacement should be considered an inherited asset and only the <u>incremental</u> training costs imputed to the replacement.

Not all C/E studies within the Army have considered the time phasing of costs, which may differ among alternatives and have an important influence in decision making. An early study by RAC (4) on our tank program did an excellent job in presenting the cost versus time consequences for each alternative, breaking out developmental, investment, and O&M costs. There is a continuing need for such a thoroughgoing approach. Associated with differing prospective expenditures over time is the need to use discounting and the need to observe budgetary constraints. We feel that questions bearing on these issues should be settled early in the analysis in concert with decision and policy making. Arbitrary discount rates are unwarranted.

For high-level C/E studies, system life-- on the basis of which costing is performed-- is considered to be 5 or 7 years. This is probably realistic for some systems, but may under-rate others badly. In the area of weapons, we feel that alternative, competitive systems should be assigned different lives if that seems warranted. Alternatives can then be compared on the basis of average cost per year of operation.

For many "conventional" Army systems the end of life value--whether value on the civilian market or as an asset to a new system--may not be negligible and should be considered in costing Army systems. Unfortunately, this has not always been done.

There are many problems that are common to the evaluation of both system cost and system effectiveness. Briefly some of these are:

a. questions of commensurability in the treatment of components of cost and components of effectiveness.

b. questions pertaining to the uncertain utilization of the system such as

1. extent of wartime use

2. nature of conflict

3. mobilization

c. questions pertaining to the rate of obsolescence of the competing alternatives

d. treatment of uncertainties in development, manufacture, and maintenance.

These common problems pose some of the greatest difficulties in structuring and conducting C/E.

We mentioned earlier that commensurability was achieved among effectiveness components by attempting to weight the components by factors derived from tactical military experience. In view of the changing nature of war

and the subjective nature of human experience, we are quite skeptical of the validity of this procedure and do not recommend it. In fact, if commensurability were only achievable by this means, we feel it is preferable to present the components effectiveness to the decision maker separately. His will then be the responsibility of attaching value to each. Also there are instances where differing resources such as operating personnel and plant should not simply be assigned a dollar value and aggregated for each of the alternatives, particularly if they differ markedly in emphasis among alternatives. Obviously a compromise must be made between complexity or detail and aggregation. It is of value to present the decision maker with a simple picture, omitting detail where this can be done; however, when a multidimensional concept such as effectiveness is given a single number in the interests of simplicity, meaning is lost.

Clearly, in the absence of any deterrence effect, a military system has value only when it is used tactically. (Systems with no prospective tactical use are not funded regardless of their potential combat effectiveness.) If it is true that the amount one is willing to pay for a military system is related to the expected military benefits or value, then it follows that the decision to acquire a system is based upon rational estimates of future tactical use. For costing purposes as well, it is necessary to make an assumption about the extent of future conflict, since generally wartime and peacetime costs of system operation are not the same. Therefore, any C/E study must come to grips with the nature and extent of future conflict. Few Army studies have!

Perhaps because of the pressure to complete a C/E study, not enough use has been made of parametric and contingency analyses within the Army. These are methods which we feel are useful in approaching the uncertainties in development, manufacture, operation, and maintenance faced in both cost and effectiveness analyses.

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COST EFFECTIVENESS PROJECTS IN THE DEPARTMENT OF THE NAVY

The Navy, like her sister services, recognizes the value and, in fact, the necessity of examining the economic as well as the operational consequences of choosing between competing programs. The demand continues for a high degree of availability in weapon systems and supporting resources. This requirement coupled with budgetary restrictions provides the impetus for utilizing such techniques as cost/effectiveness analyses. Although in their early developmental stage, these will be undoubtedly only the first generation of such evaluative methods at the disposal of the decision-maker. With this thought in mind, we will review some of the specific work on effectiveness today in the Navy. Before proceeding further, it is well to mention that the studies and models cited here are but a few of those currently underway in the Navy today. In addition, the scope of this paper is not limited to only that work which treats of cost/effectiveness in a strict sense of the words, but includes as well those studies, projects or tasks which concern themselves in some way with evaluating performance together with its attendant costs. Other terms such as "readiness," "systems effectiveness," and "logistics support performance" were uncovered in the course of this survey and are often used interchangeably in the decision process aimed at answering such questions as:

1. Which of several competing weapons systems should be developed with available resources?

2. What should the logistics support policy for a ship or a system be to provide the maximum material readiness?

3. What are the consequences in terms of performance of a budget reduction?

The thread of commonality that is woven through each of the following studies is the project's concern with the ability to measure a system's performance or effectiveness in accomplishing its designed function and to relate this performance to some expenditure of resources.

The following is a list of projects:

Cost Effectiveness Projects in the Navy

- 14 Classified CNC Projects
- Generalized C/E Models
- FLSIP
- METRI
- LMI Project 6 P (WASO)

A number of studies on which cost effectiveness analysis is being carried out are of a classified nature. The extent to which they can be discussed is limited to an identification of their titles. These are shown here merely to identify the general areas of operation and scenario situations which prompt the need for cost effectiveness evaluations. Most of these, as you can see, have logistic support connotations.

Chief of Naval Operations Projects

1. Strategic Implications of Overseas Bases and Island Possessions.
2. Nuclear Power for Surface Warships Endurance.
3. U. S. Merchant Marine Shipping Levels.
4. ASW Force Levels (Cyclops II).
5. Non-Nuclear Ordnance (Combat Consumables).
6. Sea-Based Logistic Support

7. Role of Nuclear Offensive and Defensive Forces (1970 - 1980).

8. Reserve Fleet Evaluation.

9. Logistic Support of a Land Campaign.

10. Control of the Sea (Cyclops III).

11. Petroleum, Oil and Lubricants Studies

12. Communications, Command and Control.

13. Amphibious Warfare.

14. Logistic Support of Navy Combatant Forces

C/E Generalized Models

The Navy is becoming more and more oriented to the process of thinking and decision making which embodies the principles of systems effectiveness and cost evaluation. System effectiveness which is becoming a popular topic in the offices of the Chief of Naval Material is defined as "the capability of a system to operate successfully when required under specified conditions or environment." The basic equation representing system effectiveness is the $E_s = PAU$, where E_s is system effectiveness, "P", the system's performance; "A", the availability or the fraction of time the system is ready to function; and "U", the utilization or fraction of the performance capability actually utilized for a specific application in a specified environment.1/

The term P - Performance is a numerical index expressing the system's functional ability assuming a hypothetical 100% availability, reliability, and utilization of performance capability in actual operation. This index can be expressed in any suitable terms dependent upon the nature of the mission. For instance, in a satellite system it could be in rated pounds of

1/ Systems Effectiveness Presentations, compiled by the Systems Effectiveness Branch of the Office of Naval Material, January 1965.

payload into orbit per successful vehicle. In a missile system it could represent target square miles "killed" per successful launch. In effect, it is the mission determinant parameter.

The term U - Utilization is the fraction of the performance capability actually utilized due to the specific application and the environment encountered. In effect it expresses all of the effectiveness degradation due to causes external to the system itself. This term is largely out of the control of the designer and one which in the final analysis can only be established ex post facto. Nevertheless, it is a function which can vary as a result of inherent design limitations. Such factors as consideration of operational, physical, and in some cases political environment factors operates on this function.

The central factor "A" is the period, or fraction of time, that the system is ready and capable of fully performing its mission. Conventionally "A" is expressed as:

$$A = \frac{MTBF}{MTBF+MTTR}$$

where MTBF is the mean-time-between-failures.

MTTR is the mean-time-to-repair.

Of course, in obtaining systems effectiveness, we must pay out or incur a cost. When we insert the element of cost, we add another dimension to our evaluation. The form of the expression now becomes, $E_c = \frac{PAU}{C_a + C_u}$ where, E_c is the cost effectiveness of the system in question, where C_a is the cost of acquisition and C_u, the cost of utilization. The cost of acquisition (C_a) is the total dollar cost of development and production, while the cost of utilization is the average annual dollar cost of operating and maintaining

the system, including the external cost of its failures multiplied by the number of years of useful life. The following is a list of all of the cost factors which comprise the two cost terms in the denominator of the C/E equation.

C_a (Cost of Acquisition)

Development
(Ops Analysis
(System Definition
(System Design
(Hardware Design
(Test Evaluation

+

Production
(Procurement
(Manufacture
(Installation
(Test
(Training

C_u (Cost of Utilization)

Operations
(Personnel
(Facilities
(Utilities
(Special Inputs

+

Maintenance
(Personnel
(Facilities
(Spares
(Logistics
(Diagonistic Aids

+

External Cost Due to Failures

In the past we have had a tendency to look only at the costs of procuring or acquiring a system in deciding on a major development and eventual procurement action. However, the cost of utilization is, at least, as important. We can no longer afford to put a system into the hands of our operating forces which is too costly to operate and maintain.

Another addition to the E_c equation that has been proposed is the inclusion of a military worth, parameter W. With it, the new expression for E_c reads:

$$E_c = W \frac{PAU}{C_a + C_u}$$

W is defined as:

$$W = \frac{(f_1 w_1 + f_2 w_2 + \ldots f_n w_n)}{n}$$

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where f is the fraction of the system effort devoted to a given mission element w, and the $\sum_i f_i = 1$

An additional consideration that could be incorporated would be the degrading of military worth over time - this degrading of w over time would cause the E_c expression or as it might now be called, E_d (Defense Effectiveness) to look like:

$$E_d = \frac{W}{E_t} \frac{(\ PAU\ )}{(C_a + C_u)}$$

W and E_t in this equation are both pure judgment factors and would require gaming or similar type of simulation to provide an insight into this value to the analysis.

One point to bear in mind regarding the C/E expression is that the value of C/E is an index. As described here it is useful, then, as a relative measure or means of comparing competing candidates for procurement.

One of the difficulties encountered in using this as well as most of the other cost models is the availability of sufficiently reliable and representative cost data. Another difficulty is relating properly cost factors which are not of a commensurable nature. As a gross means of looking at two separate systems, this type of model would appear satisfactory, however, in any particular case, an error in one of the cost parameters could lead to an incorrect choice. We have a long way to go in refining the systems which currently provide us with cost information. Another point to make here is that we are accustomed to thinking of cost as being dollars. However, dollars are only the medium of exchange. The resources money represents are: manpower, material, facilities, and time. We must, therefore,

choose between these in determining which resources to expand in gaining system effectiveness. Therefore, cost effectiveness is really a measure of how well we spend these resources in obtaining our objective.

FLSIP

The Fleet Logistics Support Improvement Program (FLSIP) was initiated by the Chief of Naval Operations under an OPNAV Instruction 4441.12 of 27 August 1964 to set forth basic Navy policy governing the determination of fleet material requirements, fleet asset distribution, and for prescribing the shipboard endurance necessary to achieve the desired standards of logistic readiness and endurance. Basically, the objective is to determine the range and depth of repair part and equipment - related consumables necessary to provide individual ships with 90 days basic combat endurance. The Bureau of Supplies and Accounts, together with the Bureau of Ships and Bureau of Naval Weapons, is developing explicit rules for determining which candidates are to be selected for stocking aboard combatant hulls. The procedure being set forth is to divide candidates for stocking into demand based or into insurance categories. For items in the insurance category, questions are asked concerning their vital or non-vital effect on parent assemblies. This rough determination of essentiality has the tendency to lower the range of items carried for insurance protection. As a consequence, there can be significant reductions in shipboard allowance list investment.

The specific rules to be used are stated as follows:

For the thousands of parts that are candidates for a shipboard allowance list, the maintenance code is used as the range selector. That is, an item or part is not considered for stocking unless the maintenance code indicates

a shipboard repair capability. After passing the candidacy test, the shipboard population of the item and its expected replacement rate is used to categorize it as a demand based item or as an insurance item. If the product of shipboard population times the expected quarterly (90 days) replacement rate is ≥ 1, the item is classed as demand based and is stocked at a 90% protection level for 90 days. If the product is < 1, it is classed as an insurance item and a series of two questions concerning its vital/non-vital nature are posed. The first asks whether the component to which the particular item belongs (part) is vital or not to the performance of the ships mission, and, secondly, whether the part itself is vital to the performance of its parent component. The insurance items which carry the following vital (V) / non-vital (NV) designation are excluded from the allowance list.

	Part-to-component		Component-to-hull
1.	V	———	NV
2.	NV	———	V
3.	NV	———	NV

In other words, if an item is given a vital/vital (VV) code, it is stocked (under the additional condition that there has been at least one demand for it somewhere in the Navy system over the past 2 years).

There relatively basic rules have been tested on several ships and will be implemented fleet-wide in the near future. Current expectations are that the use of these stocking criteria will reduce the shipboard range of line items by about 25% with a similar reduction in dollar investment. The

use of these rules represents the first extensive application of historical usage data (EDRF)[2] and military essentiality considerations in allowance list decisions.

The one area which might be cited as a shortcoming in this technique is the exclusion of explicit cost parameters in the stocking criteria. A technique which does incorporate the price of repair parts in its allowance determination is a project on the horizon, entitled METRI, which is currently in the research and development phase.

METRI

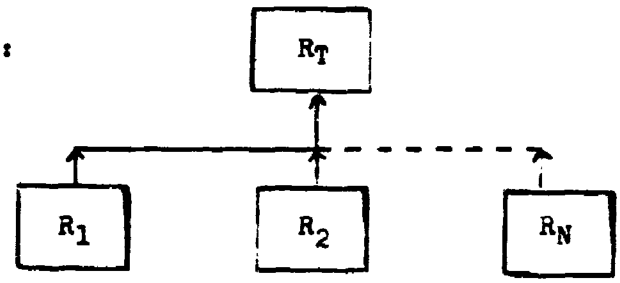
The Military Essentiality Through Readiness Indices (METRI) project under the direction of the Bureau of Supplies and Accounts (BUSANDA) has as its objective the development of a technique for determining shipboard allowance list stocks. Unlike FLSIP, which will soon be utilized throughout the fleet, METRI is still undergoing research and development. As currently formulated, METRI involves the development of an engineering model that functionally describes the operation of a force unit such as a ship. The model is a series of charts that shows how each item in the force unit contributes to its overall mission. When the model is completed, it is converted into a set of equations which are then computer programmed and used to simulate the effect of failures of parts installed aboard ship. In this way, the effect of each individual item in the force unit can be traced in a system-wise fashion. The simulated model reproduces in a short period of time experience that would take many years of conventional operation to obtain.

2/ L. Minuough, AIRAND REPORT 39,
Experienced Demand Replacement Factors (EDRF),
U. S. Navy Ships Parts Control Center,
25 January 1963

The model combines military judgment with engineering data to represent the functional interrelationships of parts-to-components, components-to-equipment, equipment-to-system, etc. aboard a ship and to integrate the decisions on what items to support rather than treating each type of part individually on an item-to-item basis as is currently done.

The ship is modeled by using the four following relationships:

<u>Supplements</u>:



$$R_T = \left[K_1 R_1 + K_2 R_2 + \ldots K_n R_N\right]^A$$

R_T = Net system readiness

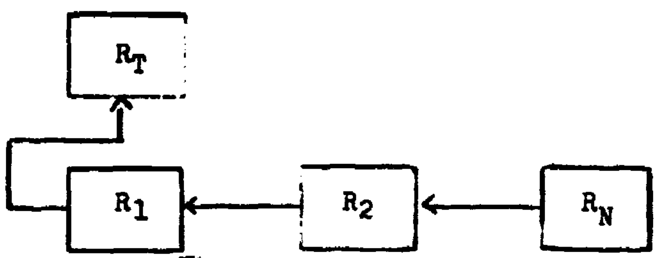
R_1, R_2, ... R_N = Individual readiness of each of the supplementary elements

K_1, K_2, ... K_N = The relative importance or contribution of each element

where: $\sum_{i=1}^{N} K_i = 1$

A = A constant indicating how the system readiness is affected by element readiness

<u>Series</u>:

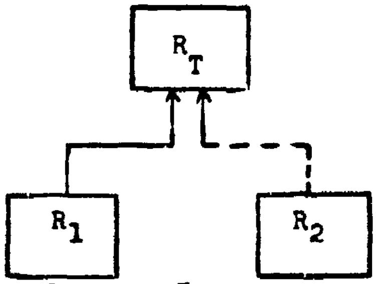


$$R_T = \left[R_1 \times R_2 \times \ldots R_N\right]$$

R_T = Net system readiness

R_1, R_2, ... R_N = Individual readiness of each of the series elements

Alternate:



$$R_T = R_1 + \left[(1-R_1)\ KR_2\right]$$

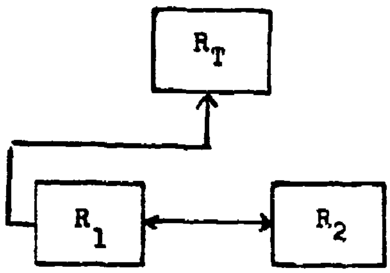
R_T = Net system readiness

R_1 = Readiness of the primary system

R_2 = Readiness of the alternate system

K = Contribution of the alternate system when acting in place of the primary system

Collateral



$$R_T = R_1 \left[K + (1-K)\ R_2\right]$$

R_T = Net system readiness

R_1 = Readiness of the essential element

R_2 = Readiness of the desirable, but not essential, element

K = Percentage of the rated performance that can be expected if the system must operate using only the essential element

The following equation is used with the model structure of the ship to determine the ΔR or:

$$\Delta R = \frac{1}{m} \left\{ \left(\frac{m-1}{m}\right)^{-(n+1)} e^{-\lambda T} - e^{-m\lambda T} \sum_{k=o}^{n} \left(\frac{m-1}{m}\right)^{k-n-1} \frac{(m\lambda T)^k}{k!} \right\}$$

where: m = number of applications aboard ship of the i^{th} part.

n = number of spares for all m applications of the i^{th} part.

λ = failure rate of part.

"change in material readiness" that results from adding additional units of a repair part. By tracing the path from the part level to mission level of the ship, a ΔR can be determined for each part. The $\Delta R/_{\Delta P}$ is computed for each item where ΔP is the unit price. The $\Delta R/_{\Delta P's}$ are sequenced in descending order and the list is truncated at any desired investment level. The range and depth of items so determined constitute the allowance quantities for the particular investment level. By computing a number of such lists, a readiness vs. cost curve can be constructed.

Current work in developing a technique for deriving consistent and operationally meaningful judgment factors for the model is being carried out. The computational approach is under revision to provide for a technically sound and operationally feasible tool for allowance list decisions. One of the problems confronting this as well as other projects requiring comprehensive and accurate input data is the lack of such information. The Navy's 3M Program (Standard Navy Maintenance and Material Management Program (SNMMMP)) shows promise of generating useful failure rate information several years from now.

Returning to more standard type cost effectiveness approaches, the Weapons Systems Analysis Office (WSAO) of the Bureau of Naval Weapons collaborated with the Logistics Management Institute (LMI) during the latter's development of improved methods for evaluating alternative logistic support programs for major weapon systems. The purpose of WSAO participation was to assist the F-4 Project Manager in providing the input data required by LMI for this task and to pursue jointly with LMI the development objectives of the task.

The means for making the comparative evaluations of alternative support programs are provided through the use of a "ratio" factor (R) and a "trade-off" factor (T). The "ratio" factor R was developed primarily for use at OSD and Military Department levels and is a measure of a Support Program's Effectiveness. The expression for this resembles the generalized models described earlier.

$$R = \frac{U + kD}{C_A + C_s + C_o}$$

where: R = Support Programs Effectiveness for a given Weapon System

U = Up Weapon System Hours; a function of total program cost

D = Down Weapon System Hours

C_A = Cost of Acquisition

C_s = Support Cost (personnel, technical data, etc.)

C_o = Cost of Operator

k = A constant modifying down-time hours

The R factor can be used to calculate the best C/E quantity of an individual support element where the relationship between up-time and the

cost of the element is known. The k^{th} support item quantity is determined by:

$$R_k = \frac{U_f\ (N_k)}{C_1 + N_k C_k}$$

where: R_k = $^C/_E$ ratio for determining best quantity of the k^{th} item

$U_f(N_k)$ = Up time expressed as function of available quantity of k^{th} item

N_k = Number of the k^{th} item available

C_1 = Total weapon system cost except the cost of providing the k^{th} item

C_k = Cost of k^{th} item

This ratio factor (R) was designed for support program analysis involving complete fleets or inventories of weapon systems.

The trade-off factor (T) was developed to make the $^C/_E$ concept more usable at lower levels of responsibility. "T" represents a weapon system's (aircraft, tank, etc.) dollar cost per hour i.e.,

$$T = \frac{\text{Programmed acquisition cost}}{\text{Number of available hours during planned service life of system.}}$$

Given several support alternatives, the T factor is calculated and used in the following expression:

$$NDB = \Delta U \times T - C_M$$

where: NDB = Net Dollar Benefit

ΔU = Predicted increase in Uptime as result of support alternative (e.g. modification to the weapon system)

T = Trade-off factor for system

C_M = Cost of support alternative

<u>EXAMPLE I</u>

	ΔU (given)	C_M (given)	T (Calculated)	NDB (Calculated)
MOD A	1×10^5 hrs.	$\$4_M$	$50/hr.	$\$ 1_M$
MOD B	5×10^4 hrs.	$\$2.7_M$	$50/hr.	$\$-.2_M$
MOD C	1×10^3 hrs.	$\$ .4_M$	$50/hr.	$\$-.35_M$

From example I, it can be seen that proposed modification A is considered a good $^C/_E$ investment while proposed modifications B and C are not. The situation may be such that non-cost/effective alternatives must be implemented (i.e. flight safety, etc..), however, the approving agency should beware of the $^C/_E$ implications of the investment.

The following expression uses the above described T factor in measuring organizational support program's effectiveness:

$$R = \frac{U + kD}{T(P) + C_{s_o}}$$

where R = Organizational Support Program's Effectiveness

U = Uptime hours of system

D = Downtime hours system

T = Calculated T Factor

P = Quantity of weapons system hours during program

C_{s_o} = Total cost of organizational level support (personnel, technical data, etc.)

k = A constant modifying downtime hours

These basic tools are available to measure $^C/_E$ in a gross manner and to provide the data necessary to make more informed judgments on costly alternative actions. The need for reliable and accurate data for these type of models is obvious as is the necessity for insuring that they be kept updated.

The Navy has available $^C/_E$ models and rules described here. As mentioned, these are in varying degrees of application - from research-to-test-to near implementation. Whatever the degree of current use, the $^C/_E$ approach is gaining popularity as a measuring tool in the Navy.

William W. Klare
Lt Colonel Floyd Revere

Recent Mathematical Modeling/Cost Effectiveness Projects in the Department of the Air Force

A year has passed since the Air Force embarked on an extended formal Mathematical Modeling/Cost Effectiveness Program. A brief review of the background which lead to this extension will serve as a starting point for an illustration of how these techniques are being used by the Air Force.

At DOD's request, the Air Force Logistics Command and the Air Force Systems Command, with the assistance of the Strategic Air Command, prepared a summary of the computerized cost effectiveness efforts used on the Minuteman weapon system. Hence, successes in support planning on the Minuteman form the basis for our current research and use of simulation and mathematical models. The Minuteman planning efforts were initiated with a series of conferences held in late 1961 and 1962 for the purpose of procuring maintenance ground equipment and quantifying personnel requirements. A new technique was developed and tried for determining Minuteman Maintenance Ground Equipment (MGE) quantities in the Spring of 1962. The technique introduced on Minuteman takes into consideration three critical and frequently overlooked factors when computing manpower and equipment requirements. These are: the randomness of failure or uncertainty as to when a malfunction will occur; the workshift policy with respect to when maintenance personnel are on duty; and the point of total minimum cost or the point of marginal increase in system capability weighed against the marginal increase in cost of providing additional resources. In practical application, a set of "equal cost curves" was produced specifically tailored to the Minuteman (Chart #1). To use these curves, the provisioner had to compute the days per month the resource would spend in transit, in repair or performing its function, and the cost of the resource. Using this technique, the desired quantity from a cost effectiveness point of view can be determined and the penalty involved can be evaluated if the proper quantity is not chosen. Also, it was possible to examine the effects of failure to meet reliability, maintainability, or equipment cost estimates from both the point of view of consequent changes in requirements and of force readiness degradation.

Our presentation, describing in detail these Minuteman techniques, was given to the Air Staff and to the Department of Defense early in 1964. At this point it was determined that we should move forward with similar techniques and procedures on the F-4C and the F-111.

To centrally control the management of this effort, a Joint Command Steering Group was formed, chaired by representatives of AFLC and AFSC, with participation from the using command, augmented by members from each command's subordinate echelons. Although the Steering Group deals primarily in techniques, its objective remains the acquisition and support of a weapon through its planned service life at a total minimum cost or to obtain an economic balance between the dollars that are invested in capital equipment and the dollars that will be required to support the system.

To simplify, with little or no dollars invested in support element resources a downtime cost can be anticipated. As more resources are invested downtime cost decreases. However, a point is reached where more dollars invested in resources will not produce a comparable return (Chart #2).

The high downtime cost is caused by the weapon waiting or "queueing" for resources such as personnel, spares, and AGE. This waiting is caused by the randomness of failure and subsequent variability in demand. Under today's techniques, resources are provided based on the average demand. However, optimum resources are usually somewhere between the average and maximum demand. Using queueing theory, the point at which the optimum or desired quantity should be can be determined. On Minuteman, queueing theory was used along with reliability data to computer generate equal cost curves for use by the provisioner in establishing the optimum stock level requirement as previously discussed.

In addition to the problem of optimizing downtime costs, there are other economic considerations to be resolved using math models. These fall into the repair decision area. Is it cheaper to repair an item or throw it away? Is it cheaper to backstream repair back at the depot? The other consideration was whether we even bring repair of the item in-house. The number of reparables that would generate influences this kind of decision. If a base had a low reparable generation, the investment in base repair capability could not be amortized.

As mentioned, we launched a formal program to attempt the application of cost effectiveness modeling techniques to the F-4C and F-111. We learned early that the techniques that were used on Minuteman could not be applied directly as there was no acceptable way to determine the cost

of an hour of aircraft downtime. In the Minuteman program, $80 an hour was used for the cost of an hour of downtime. This cost was derived and used from a ten year service life compared to total weapon acquisition and operational cost.

Research then was necessary for other available procedures or models that could be used. This research revealed that General Dynamics in Fort Worth was using logistic models on the F-111, and it was found that four particular RAND developed models used the techniques that had been used successfully in support planning for the Minuteman.

With their work on the B-58 as precedent, General Dynamics is relying on several models for specific assistance during the production of the F-111 (GD/FW Brochure, MR-0-84, 18 May 1964, Description of Logistics Models on F-111 Program).

Their Subsystems Simulation Model is used to evaluate the detailed maintenance and logistics problems of a specific subsystem of the aircraft (e.g., fire control, hydraulics, flight control, etc.). This model computes the expected frequency of maintenance and downtime and the requirements for various personnel skills, equipment and facilities, and the expected removal rates of line removable units for each subsystem.

The Network Analysis Model was developed to combine the maintenance characteristics of each subsystem to describe the overall downtime characteristics of the aircraft in a perfect logistics environment (i.e., adequate spares, personnel, etc., 100 percent of the time).

The Base Maintenance and Operations Model was developed to measure the operational capability of the aircraft in its operational environment. This model utilizes the outputs from the above-mentioned GD models and combines them with flight schedules and other maintenance and operational policies to simulate the activities of the aircraft base. The principal output of this model is the operational readiness of a squadron or wing and the magnitude of the logistics problem.

The Cost Model is a computer program weapon systems cost model used to obtain RDT&E, investment and operational costs. Cost prediction equations are used to estimate costs. Computer printouts are made by specific cost categories such as spares, personnel, facilities, equipment, etc.

The Effectiveness Simulation Model measures the operational capability of the aircraft in its flight environment. The output of this model provides the aircraft availability for flying. It simulates the flight operations for up to 100 aircraft and computes airborne effectiveness. It will also accept cost inputs from the cost model for computing cost effectiveness as desired.

General Dynamics uses these models separately to analyze specific problems or together to produce a complete system analysis.

Conversely, the F-4C had no comparable effort under way and, as this system was further along in the acquisition phase than the F-111, the Joint Steering Committee decided to concentrate on the F-4C with the intent of returning to the F-111 program when our in-house competence had increased.

Along this line, then, the RAND models were applied to the F-4C by the preparation of jointly prepared TAC directive initiating data collection at the First Operational F-4C Wing at MacDill Air Force Base, Florida.

In this directive, the 836th Air Division is charged with the collection and validation of data in addition to that normally required (AFM 66-1 and AFM 65-110). One of these additional items is the "when discovered time." This is necessary to realistically separate the initial delays from the maintenance time experienced on a particular failure. Others are: "when work started;" "delay information with reasons for delay;" "when work is completed;" "team size;" "equipment type and time of use;" "mission information;" and "operating time at failure for time change replacement and Elapsed Time Indicator installed." These data are fed by punch cards into the computers to exercise a technique developed by RAND in the Manning and Equipping area (RAND Memorandum RM-3370-PR-Manning and Equipping Decisions) (Chart #3).

This technique is designed to accurately identify the maintenance generated by operational requirements both in the amount of work and the time at which it was required. Such identification is necessary in order to evaluate flying schedules, estimate the resulting workloads and plan work schedules to minimize overtime.

Management is provided several easily read graphic displays which assist in base level action. These displays present:

Information about the time when work was first known to maintenance and when it was done.

Identification of delays of return of aircraft to operational readiness.

Summary of manpower utilization by hour/day/aircraft for personnel requirements determination and for workshift planning for any given flying program.

Recording of job elapsed times in addition to job frequency.

Computation of sortie recovery capabilities.

Examination of impact of maintenance and operations scheduling on the flight program and on operational readiness.

Although outside the purview of the Joint Steering Group, TAC has extended the use of this model to the maximum effort aircraft, the C-130 at Pope Air Force Base, North Carolina. MATS is also investigating it for use on the C-141 and the C-5A.

Using the manning and equipping technique output our next model, the base operations model or SAMSOM (RAND Memorandum RM-4077-PR-Support Availability Multi-System Operations Model) (Chart #4), is a tool for simulation of the base level activities that are associated with operating aircraft at a specified sortie schedule. For various sortie schedules, the model yields the number of sorties that were launched and provides an operational ready rate within the constraints of personnel, equipment, facilities, reliability, and malfunction rates, the time required to repair, and the parts delays that are encountered. It identifies the cause and magnitude of delays in aircraft turnaround. It should be particularly useful to the System Project Office (AFSC) and System Support Manager (AFLC) during the program definition and acquisition phase to evaluate contractor engineering proposals, determine the AGE requirements, determine their personnel requirements, and provides a base line for measuring system effectiveness in terms of the "operational ready rate," the "not operational ready, maintenance," rate and the "not operational ready, supply" rate. In the operational phase, it can be used for measuring

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the effects of proposed changes in operational plans and validate the expected resources that are needed. The model requires the use of large scale computers, relatively sophisticated inputs, and people who are thoroughly grounded in the details of the model. It requires a high level of understanding by its users.

This model has been programmed and successfully operated by representatives of the two Commands. To the degree that our personnel have learned the procedures of modeling the program can be deemed successful. However, specific instances of assistance to the F-4C program cannot be cited at this point.

While commenting on SAMSOM is appropriate to bring out that during Phase I Studies of the Program Definition Phase of the ADO-12 (Vertical Short Take-Off and Landing, or VSTOL), an advanced fighter weapon system, the SAMSOM model was used quite extensively by two competing contractors. Areas in which investigations were made were simulation of fighter interceptor operations, comparisons between alternative basing postures for tactical fighter bombers and simulation of selected operational and logistics concepts. As evidenced by its use in the ADO-12, computerized simulation is becoming an indispensable tool for evaluating new systems as they enter the program definition and acquisition phases.

The third RAND model and perhaps the most promising of the models from a long term Supply standpoint is the Base Stockage Model (RAND Memorandum RM-3644-PR-Base Stockage Model) (Chart #5). It is used to set an optimum base stock level for recoverable type items where the optimum is equal to the minimum stock value in dollars for a specified fill rate or the maximum fill rate is achieved for a specified dollar restraint. Levels are set base by base, using the base's past six months demand or in the case of new aircraft the engineered estimates of demand. The model considers the demands at base level, the percent of reparables repaired at the base, the base repair time, the depot resupply time, and the unit cost. The model assumes a zero stock level for every item at the base and then poses the question, "What is the greatest amount of fill protection per dollar unit of stock?" The model assigns a unit of stock to this item and then determines the next largest amount of protection per dollar unit of stock and so on. The method begins by stocking low cost high demand reparables and works down to low demand high cost items. The allocation process continues until the target aggregate base fill rate has been obtained, at which time the computer prints out the item and summary results.

The model was tested by RAND at Andrews Air Force Base in April 1962 on 2,802 reparable items. At that time, Andrews had 2.5 million dollars invested in base inventory and was achieving an actual fill rate of 61.3 percent. For this same dollar investment, the model projected target fill rates and measured the actual fill rates against what was targeted. If the base had been stocked according to the model, a fill rate of approximately 92% would have been achieved. The model could have achieved the same fill rate as Andrews achieved under the 2.5 million dollars for an investment of 631,000 dollars. Ogden Air Materiel Area has used the model to project their initial lay-in of spares quantities and are now working with the Tactical Air Command to make base lay-ins possibly at George Air Force Base. OCAMA is going to use the model to test the stock levels laid in for BOMARC and Minuteman. Sacramento Air Materiel Area has also run the model to project test table requirements for Category I and II tests. They are presently working with the System Project Office to determine whether or not they will use these forecasts for the lay-in of the test tables. We intend to use the model if it proves successful to make provisioning decisions for base lay-in on the F-111.

The last model, the base versus depot repair model (RAND Memorandum RM-3845-PR-Base Versus Depot Repair Decision) (Chart #6) is used to determine economic level of repair by pricing out the cost of establishing a base repair capability versus savings that would accrue from reduction in base resupply time. It considers such factors as the cost of AGE and skills, the volume of repair involved, the base repair time or the depot resupply time, the depot costs, the stock levels of end items that will be required if the item is depot repaired, and the cost of bit and piece support. It provides a display of the differential costs for the various degrees of base repair and its primary value is at the source coding conference to establish the repair level codes.

OOAMA has used the model on the F-4C fire control system. SMAMA will use it for making repair level decisions on the F-111 upon completion of the machine programming being undertaken by RAND. The evaluation will continue through the F-111 program. We hope to gain sufficient knowledge in the early phases of the F-111 to apply it to future weapons.

In summary we have been pioneering for a year, stretching the technical skills of our people and machines. Although industry to some degree and certain study and research groups have had experience in these described techniques, this is the first Air Force attempt to bridge the gap from the laboratory to the operational environment. It has not been easy nor do we expect it to ease up.

We anticipate a requirement for an evaluation of our entire program sometime in mid Summer. Pending this detailed evaluation these comments appear supportable as of today:

Success requires that the user, developer, and supporter actively participate in simulation and modeling.

Early initiation and participation are required to preclude operational experience from overrunning simulation. However, feedback data from operational experience is necessary to validate the simulation techniques.

A high level in-house competence is required to react to contractors' models outputs.

No militarily satisfactory simulation model has been identified if we define satisfactory as simple, operationally complete, automatic, realistic and multi-purpose.

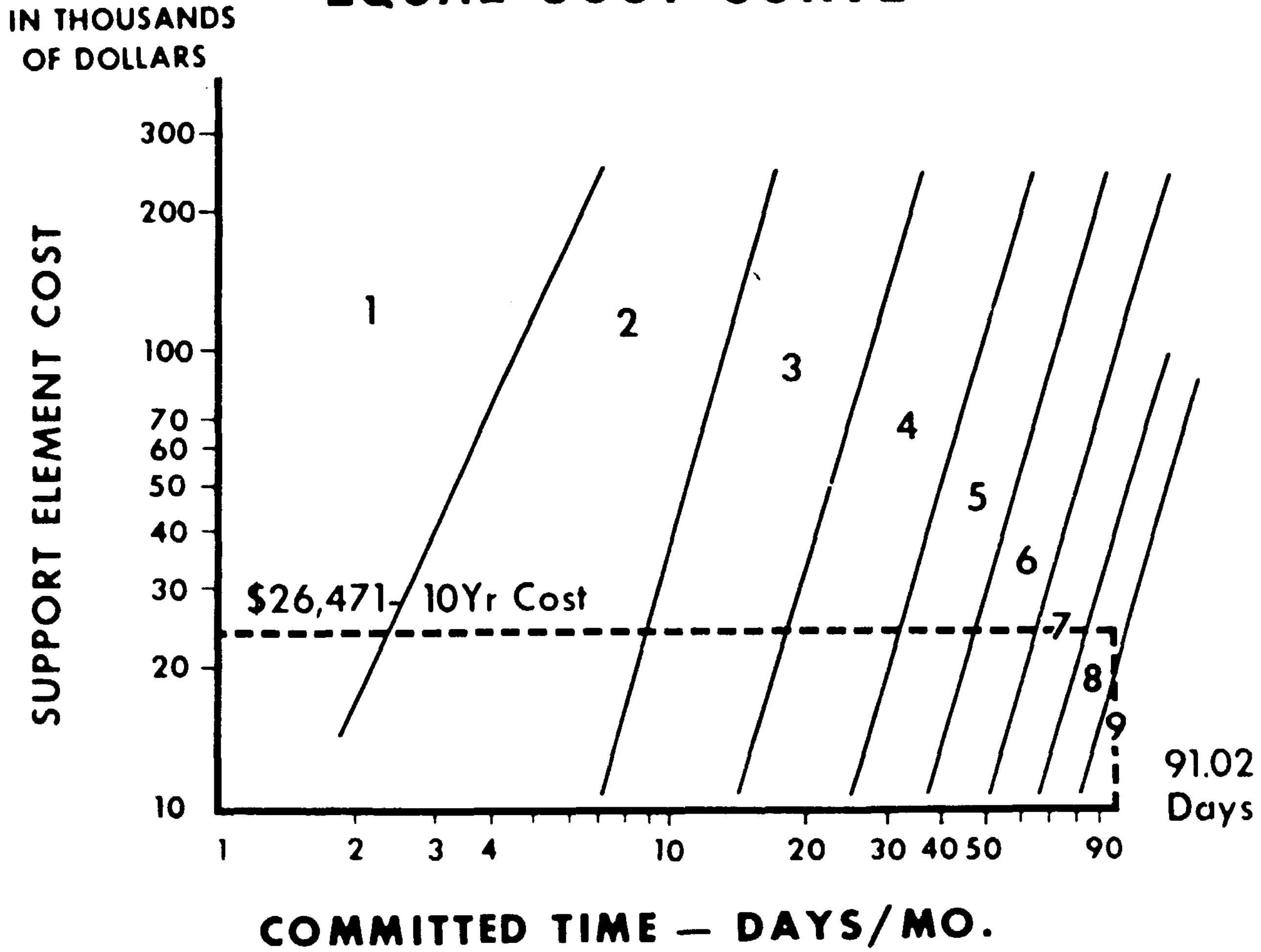
The Manning and Equipping Model provides useful data to TAC, SPO and the SSM.

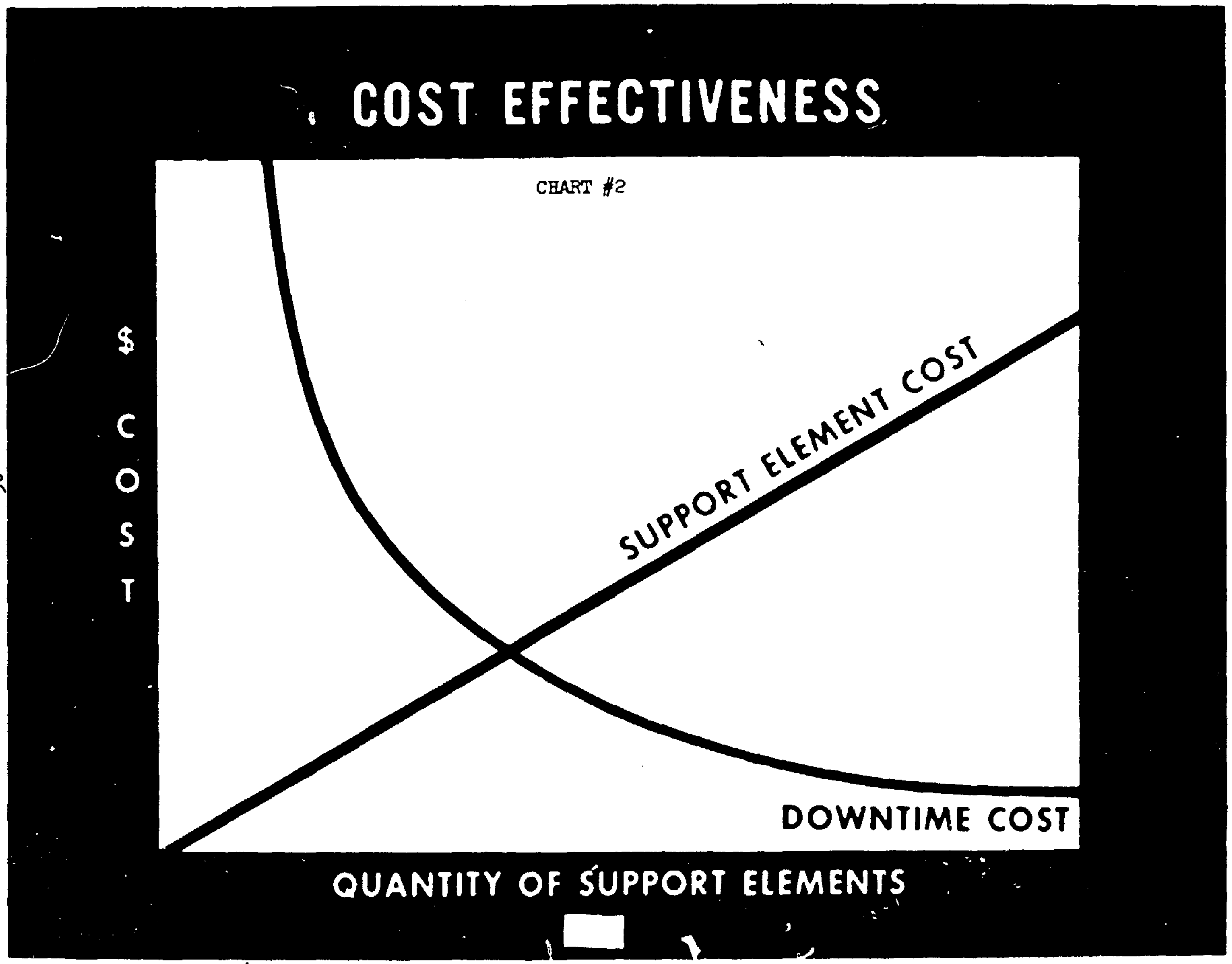
The Base Stockage Model may point the way to a more realistic and economical placement of spares at the base level.

The Base versus Depot Repair Model promises a more systematic approach to the repair level decision.

EQUAL COST CURVE

CHART #1




COST EFFECTIVENESS
CHART #2
$ COST
SUPPORT ELEMENT COST
DOWNTIME COST
QUANTITY OF SUPPORT ELEMENTS

MANNING AND EQUIPPING

CHART #3

Analysis of Base Maint Functions

Requires augmented AFM 66-1 Data

Data products to Base Maint on:

- A/C turnaround time
- Work scheduling
- Equipment use (AGE)
- Shop loads
- Personnel use

CHART #4

BASE OPERATIONS (SAMSOM)

- Simulate base-level activities
- Provides:
  - Nr. sorties launched
  - Operational ready rate

within constraints

- PERSONNEL
- EQUIP.
- FACILITIES
- RELIABILITY
- REPAIR TIME
- PARTS DELAY

CHART #5

BASE STOCKAGE

- Sets optimum BASE Stock Levels

  Fill rate vs inventory investment

- Considers
  - Demands
  - % base repaired
  - Repair/resupply time
  - Unit cost

BASE VS DEPOT REPAIR

CHART #6

- Determines economic level of repair
- Considers
  - AGE Cost
  - Repair Volume
  - Repair/Resupply Time
  - Depot Costs
  - Stock Levels (end item)
  - Bit and Piece Support

MILITARY ESSENTIALITY

by

W. H. Marlow

INTRODUCTION

Our aim is to describe some techniques for measuring military essentiality and to illustrate applications for cost-effectiveness comparisons. We suppose that we are given some definite set of military items such as a collection of equipments or repair parts, or even several different weapons systems. We suppose further that each item has some intrinsic value in the sense that its use, or availability for use, can in some way contribute to gaining military objectives. In this way we are led to the notion of a military essentiality system which will make it possible to compare the values of the items.

We will say that we have a military essentiality system whenever the following conditions are met. First, a set of military objectives must be fixed. Usually, this takes the form of a mission or set of missions. Second, a definite set of military items must be specified. This set will form the domain for comparison of military essentiality. Third, the possible means for contributing to the fixed

objectives must be known for each item. These are the attributes that are to be compared. Finally, a precedence list must be given which serves to rank all the items in order of military essentiality: 1st, 2nd, ..., last. Two different items either have the same rank or else one has the higher rank which means that its attributes are more important to the mission. In this way we obtain an ordinal system in the sense that we have determined only the relative order of worth; e. g., we may not be able to conclude that one item is twice as important as another but only that it is more important. If the precedence list serves to rank all the items on a numerical scale so that rank a is twice as important as rank $a/2$, then we have a cardinal system.

The above approach equates military essentiality to accomplishment of military objectives. In fact, the term "essentiality" could be replaced by "effectiveness" or even "readiness" with practically no distortion of meaning in present context. This is true because it is possible to select missions and sets of items so that military essentiality does correspond exactly to military effectiveness or readiness.

Our principal vehicle for exposition will be the Polaris Military Essentiality System as developed by Denicoff et al. [1]. After describing this system we illustrate various applications to allocation problems of material support, notably some involving repair parts. Our basis is the fundamental logistics model of Denicoff et al. [2]. We provide illustrations of ordinal methods wherein military essentiality is employed as a priority designator. Further examples are given employing cardinal methods wherein military essentiality is measured on a numerical zero to unity scale. We consider the joint use of effectiveness measures and cost data so that the examples deserve the label "cost-effectiveness". We conclude with a brief discussion of prospects for better military essentiality systems and improved applications.

THE POLARIS MILITARY ESSENTIALITY SYSTEM

The subject system was developed to measure various effects of failures on the Polaris weapons system. Three different levels were considered: equipment component and wearable installed part. Failures at one level were studied for their effect at higher levels culminating in consideration of effects on the Polaris mission. Five military essentiality systems were developed: a system was required at each of the three levels, one was developed to rank component-equipment combinations, and finally an overall system was derived to handle part-component-equipment-mission relations. This final system will be the only one that we need consider.

We restrict our attention to the Polaris mission: the submarine is to carry out a normal patrol cycle which will mean that there can be no external supply or maintenance support. The deployed crew must rely entirely on themselves and their available material resources in order to achieve the required states of performance during patrol.

The set of military items in question are the wearable installed parts which go together to make up the entire weapons system. We must consider parent components and equipments since the same part might have different significance according to its different installations. A common example would be a transistor which is installed in both fire control computer and in a stereo music system. Thus, the entire set of part-component-equipment combinations must be specified as the domain for military essentiality measurements.

Next, we must relate each of our items to the desired military objectives. This is accomplished in the subject system by means of questionnaires which determine precise effects of failures. Consider the equipment level. We suppose that

a failure occurs on the first day of patrol such that all installed units of the given equipment are lost with no possibility of repair. One of the choices is made to describe the mission effect: "total degradation" means that patrol would have to be terminated; one of "partial" or "minimal" is specified in case patrol would not be terminated. Two additional questions, also with three choices each are asked relative to the consequences of failure of a single unit. The second question relates to redundancy and the third to alternatives. There are a total of 27 possibilities: the highest worth equipment is one whose loss would totally degrade the mission capability and for which there is no redundant or alternative equipment available. At the other extreme, failure of the least essential equipment would have a negligible effect on the mission, and furthermore, redundant and alternative equipments exist with comparable capability to the equipment itself. A similar questionnaire is completed for each component with the feature that effect on parent equipment, equipment effect, replaces the former mission effect. Finally, two "yes or no" questions are asked concerning each installed part. Can the parent component operate satisfactorily for the entire patrol period lacking one unit of the part? Is the part installable during patrol? These are the data which enable us to determine the possible contribution by each item, e. g., each repair part, to the mission.

The actual ranking system or precedence list turned out to have 116 levels. If two items belong to the same military essentiality class (MEC), then by definition they are equally important. This system was derived systematically from certain basic principles as shown by Denicoff et al. [1]. It is of some interest to note that the original (27)(27) = 729 categories for component-equipments were reduced to 29 MEC's as outlined in Table 1.

MEC	RELATIVE ESSENTIALITY
116	Highest
115 - 94	High
93 - 91	Intermediate
90 - 88	Low

Table 1 - MEC code assignments

ORDINAL METHODS

In this section we illustrate the use of the Polaris MEC system where the precedence list is used in ordinal fashion only, e.g., as a priority designator. We will phrase our examples in terms of allowance lists. Such a list specifies the repair parts which must be carried on board ship. Future demands for the repair parts are uncertain and there exist limited stowage space and limited funds for allocation. We will use MEC values to indicate effectiveness while costs are expressed in cubic fee, in dollars, or in some combination. We consider future demands for a repair part to be represented by a random variable X. For the time period of interest, say one patrol period for a Polaris submarine,

$$\Pr\{X = i\} = f(i), \quad (i = 0, 1, 2, \ldots).$$

Then the value, $F(k) = \Pr\{X \leq k\}$, of the cumulative represents the probability that demand will not exceed k units. We will assume that demands for

different repair parts are independent. Different repair parts will in general have different distributions so that where necessary we will use $F^{(j)}$ for the distribution associated with the jth part. It turns out that there are about 2000 items in MEC 116 for a Polaris submarine together with all of its weapons systems. By definition, each of these items is an absolutely essential repair part. In case there is a failure requiring replacement of such a part and there are insufficient units in stock, then the submarine must terminate patrol. This leads to

Example 1. Because of the absolute essentiality of repair parts in MEC 116, we argue that these parts should be given overriding priority. Effectiveness of repair part support requires that there be practically no shortages of such parts. Expected overall effectiveness of the weapons system could be analyzed with the result that a minimum threshold, say 0.99, is specified for the probability that demand will not exceed supply for any of the, say, n items in MEC 116. This can then be immediately transformed into an equation for determining the allowance quantity to be placed on board for the jth item. That is, recalling that the items are assumed to be independent, we could use the following procedure for a uniform allocation of readiness over the n different parts. We stock k_j units of the jth part according to

$$k_j = \min_i \left[F^{(j)}(i) \geqq t \right] \quad \text{where} \quad t = (0.99)^{1/n}.$$

In words, t is a critical threshold for each repair part in MEC 116; each part is stocked in quantity such that the probability that demand will not exceed supply equals at least t. If n = 2000, then t = 0.999995; however, in practice t can be chosen somewhat smaller, e. g., 0.999990 with the result that the product of the 2000 $F^{(j)}$ will still exceed 0.99. The reason is simple: for each j, $F^{(j)}(k_j)$ is generally strictly greater than t.

In this example MEC 116 has been treated with overriding priority because of its significance for effectiveness of the weapons system. Cost considerations are not used except in a general way to limit the allocation of space and funds; these resources are simply expended as necessary prior to allocation to the other MEC groups.

Example 2. Let us consider MEC groups 115, 114, ..., 94 which constitute the remainder of the "high worth" classification. We could do something similar to the procedure for MEC 116 and let t be set at an appropriate value for each MEC group. However, it turns out to be more promising to let t vary, item by item, as a function of MEC, unit packaged stowage requirement or cube in cubic feet, and unit dollar value or price in dollars. We will furnish details below in Example 3. At present we wish to use the MEC code as a priority so as to pre-empt resources in some manner less severe than in Example 1. Two cases are worthy of note. First, we could argue that there should be at least one unit stocked of each repair part in MEC 115, 114, ..., 94. This could be called an MEC override. Second, we might wish to specify a minimum probability threshold, e.g., a minimum protection of t^*, so that $t' = \max(t, t^*)$ would be used where t is the "normal" function of MEC cube and price. Some effectiveness considerations which could lead to this latter procedure are illustrated by repair parts for certain sub-systems for which minimum performance probabilities are specified. For example, there may be 50 parts spread over 115 to 94 for which a certain joint probability determines the minimum protection t^*.

CARDINAL METHODS

Let us suppose that we are interested in applying a given military essentiality system. Then, although we are willing to retain the precedence list, it may turn out that actual magnitudes must be assigned to military essentiality. This would

happen if some mathematical model were to be used that required a single numerical scale for both effectiveness and costs. The nature of the application and the details of the original essentiality system would of course have to be considered along with the model before cardinal values could be assigned. Indeed, it would be rare that a numerical scale could be chosen for essentiality independent of the details of proposed application. Something similar often applies to costs, even in dollar values, which must be standardized or rescaled for use in a numerical application. These points will be illustrated in

Example 3. We consider a simplified version of the Polaris model developed by Denicoff et al. [2]. The specific case we present is an adaptation of the classic "newsboy problem". This problem concerns the boy who is required to buy his papers at 2 cents and sell them at 3 cents, and is not allowed to return his unsold papers. Assuming that customers will buy his papers according to a given probability distribution, the problem is to maximize expected profit.

Our example will start with a single repair part candidate for placement on a Polaris allowance list. As the counterparts of the 2 cents and (3 - 2) cents, respectively, we have the following two positive numbers specified relative to a single patrol period.

A = penalty per unit stocked in excess of number demanded during the entire patrol.

B = penalty per unit demanded in excess of number stocked for the entire patrol

We suppose that demand for the item will be represented by a discrete probability distribution F as in our earlier examples except that now we require that F

have a finite mean. Then, for each number $s = 0, 1, 2, \ldots$ our expected loss in stocking s units equals

$$L(s) = a(s)\,A + b(s)\,B$$

where

$$a(s) = \sum_{i=0}^{s} (s-i)\,f(i) \quad \text{and} \quad b(s) = \sum_{i=s+1}^{\infty} (i-s)\,f(i)$$

are the expected surplus and shortage, respectively. It turns out that there is an optimal quantity to stock equalling

$$n = \min_{s}\,[F(s) \geq t]$$

where

$$t = B/(A+B)$$

is a critical probability threshold similar to some appearing in our earlier examples. We notice that t is determined by the value of the ratio B/A so that it would suffice to specify the relative value of B to A. However, it will be convenient to proceed as though A and B are to be determined separately.

The object in our present example is to select numerical values for the surplus or holding cost, A, and the shortage cost, B. Our item in question is described by a unit packaged volume or cube, c, expressed in units of say 10^{-4} cubic feet and a unit price, p, expressed in 10^{-2} dollars, i. e., cents. In addition, it has an assigned MEC, w, which is one of the numbers 116, 115, ..., 59. (We stop at 59 so as to restrict attention to repair parts that can be installed on patrol.)

Let us consider the surplus cost A. In the case of a Polaris submarine

the critical resources are stowage space and dollars. One possibility for assigning A is to use a weighted sum of cubic feet and dollar value. Since A is to be positive, it is convenient to eliminate zero values from data errors by defining

$$A = \max[(\gamma c + \pi p), 10^{-10}]$$

where γ and π are positive scaling factors to be discussed below.

The unit shortage cost B should certainly depend on MEC. However, there are cogent reasons for letting it vary with cube and price as well. One useful choice is

$$B = \max[(e^{-\alpha(116 - w)} - \gamma c - \pi p), 10^{-10}]$$

where α is a scaling factor for MEC. An exponential is used for several reasons. First, it is a convenient way of establishing a scale with a maximum military essentiality value of unity which diminishes essentially to zero. Second, military value falls off sharply and "reasonably" as w decreases from 116. The rate of decline between any two points depends of course on their values so that the curve flattens out as lower MEC values are reached. Such "exponential decay" is the determining factor in our choice. We have subtracted $A = \gamma c + \pi p$ from the exponential so as to take credit for the cost we avoided by not stocking one unit. Looking at this another way, we now have the sum $A + B$ expressed as a function of MEC alone; this corresponds to using MEC as the sole determinant of intrinsic worth in the present application. Finally, our definition of B taken together with that for A serves to guarantee

$$0 < B/(A + B) < 1$$

so that $t = B/(A + B)$ can be used as a probability threshold.

Let us defer discussion of the scaling factors γ, π and α until we have indicated how we would compute an entire allowance list. The first step in such a computation would be to arrange the items in "priority" order of nonincreasing essentiality, starting with the $w = 116$ items. Then for fixed γ, π and α we would compute n for each item in order, at the same time accumulating total cube and total price. This procedure would correspond to optimizing for each item individually without any restrictions on total cube or total dollar value over all items. When we reach the end of the list after the first time through, we have accumulated total cube C_1 and total price P_1. These totals may or may not be acceptable. For simplicity, let us suppose that overriding significance is attached to stowage space and that a limit C^* will govern: in order to complete an allowance list determination, we must find a list representing C^* cubic feet. After the first pass, if $C_1 = C^*$ we are finished. If $C_1 < C^*$ we would diminish some or all of γ, π and α and make a second pass terminating with C_2 cubic feet. If $C_1 > C^*$ we would increase some or all of γ, π and α and try again, etc. Exact rules would have to be specified relative to definite objectives for a particular set of items. For example, if "always" $C > C^*$ then we may have to stop computing midway in the priority list with that item for which total cube first exceeds C^*. This would mean that items below this cut-off item would not be considered. Experience at the Logistics Research Project has shown that it is quite easy to determine empirically some ranges for γ, π and α which permit wide choices of types of allowance lists (e. g., wide ranges in terms of varieties of items stocked with $n \neq 0$ vs. relatively narrow range stocking mainly high MEC items in large quantities each, etc.). In addition, it is generally true that calculations do not have to be terminated before all items can at least be considered for placement on board. However, in the present paper we will not dwell further on procedures and rules for computing allowance lists.

Empirical work at the Logistics Research Project has had the following illustrative results. A reasonable value for α for Polaris allowance lists has been $\alpha = 0.10$ as summarized in Table 2 where "scaled essentiality" denotes $e^{-\alpha(116 - w)}$

MEC	SCALED ESSENTIALITY	INVERSE ESSENTIALITY
116	1.00	1.0
108	0.45	2.2
93	0.10	10.0
59	0.003	300.0

Table 2 - MEC scale for $\alpha = 0.10$

Since cube is the primary constraint, it has been found that choosing the price scale with $\pi = 10^{-6}$ permits wide latitude of total cube. A representative Polaris allowance list would use γ near 10^{-3}.

Tables of allowance quantities as functions of MEC, cube, price and F are contained in the U. S. Navy Special Projects Office Instruction [3]. These tables incorporate the parameters specified just above together with some special rules along lines described in Examples 1 and 2. In addition, application of similar overall procedures for other echelons of supply are described in the Instruction.

In summary, our present example illustrates the use of cardinal values for each of military essentiality, cube and price together with a loss minimization model. An overall cost-effectiveness procedure is employed whose structure is quite complex. Even for a single item, the optimum quantity depends on the associated probability distribution together with the three variables, and their scaling factors, which enter computation of

the threshold $t = B/(A+B)$. Military essentiality enters in particularly vital fashion as an overall adjustment factor. If we return to Table 2 and consider MEC 93 and the right-hand entry 10.0 we may make the following observation. The number 10.0 does in some sense mean that an item in MEC 116 is 10 times as essential as an item in MEC 93. However, it does not mean that an item in MEC 116 will be stocked in quantities tenfold larger. Instead, it means that the exponential scale will be this much larger. For example, consider items alike in every respect except MEC and for which

$$\gamma c + \pi p = 0.002.$$

Then $t = 0.996$ for MEC 116 and $t = 0.980$ for MEC 93.

PROSPECTS

The methodology described above for measuring and applying military essentiality has proved to be of great value for practical applications within the Navy. There are many ways in which these procedures might be improved. In the present section we give brief attention to a few specific points.

Considering the Polaris Military Essentiality System itself, there would be interest in extending it so that additional precision could be applied to failures at low and intermediate MEC levels. For example, how should on account for effects of simultaneous failures on weapons system effectiveness?

There are, of course, corresponding problems for groups of weapons. Should new military essentiality systems be developed or can we adapt the individual systems? In the case of the Polaris weapons system should we try to apply the submarine MEC system at the tender or depot levels?

Studies are underway at the Logistics Research Project aimed at obtaining answers to questions such as those posed just above. Work is also

in progress on developing new techniques for handling effectiveness models, e. g., new loss functions to be minimized, in order to improve the usefulness of solutions to resource allocation problems. In conclusion, we would report that prospects seem excellent, not only for improving the measurement of military essentiality, but for applying it to significant military problems which heretofore have resisted practical solutions.

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COST EFFECTIVENESS FOR

INTEGRATED LOGISTICS SUPPORT

SYSTEMS & EQUIPMENT

(DOD DIRECTIVE 4100.35)

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More and more an urgency exists at all levels of management to improve our day-to-day decisions and to improve our forecasts of probable future events. This is particularly true in the Department of Defense where present and future weapon system programs and other major program changes are evaluated. Many of these decisions are very broad and complex and involve high national risks in military effectiveness. They generally must be made against a backdrop of world uncertainty and rapid change. Often even very small improvements gained may represent substantial sums of defense dollars.

While gross errors in decision logic are usually caught by the intuitive-hueristic methods now inculcated in our defense and industry management at all levels, it remains for improvements in new scientific management techniques and new information systems to detect and perhaps eventually correct or prevent some of the more subtle errors in judgment. In my opinion, the sum of these small improvements toward increasing our overall productivity may eventually mean the difference between "burying" the enemy and in being "buried."

I have been asked to discuss with you today the experiences we have gained from the efforts of a joint DOD-Industry task group organized recently to study cost-effectiveness and other similar techniques. The primary objective of the study was to enhance implementation of DOD Directive 4100.35 which requires the "Development of Integrated Logistics Support for Systems and Equipment." In my presentation, I will cover three (3)

principal areas - First, a bit of background leading up to the study; Second, the "Modus Operandi" employed by the study group; and lastly, a brief discussion of the findings and conclusions of the group to date. I should like to point out that the study is in its final phases and any recommendations I may suggest will only be tentative at this point.

The background for the task group efforts began in June of 1964 when OSD issued DOD Directive 4100.35. This directive requested each of the Military Departments and appropriate defense agencies to develop integrated logistics procedures in support of systems and equipment. The directive placed emphasis on two principal concepts - One, that the plans for employment of a new system must be in-balance with the plans for its support; and two, that the logistics implications of that system must be considered at the earliest possible time in its life cycle and be integrated as one of the principal elements of trade-off, along with the implications of research, development and test. These concepts may almost sound like truisms, but let me say we are not talking about integrating one plan, we are talking of literally dozens of functional plans and many hundreds of variables.

In late July of 1964, the DOD Maintenance and Equipment Council, composed of the senior military and civilian logisticians of the Defense Department and their supporting Ad Hoc Committee, assigned nine (9) specific development tasks to the Military Departments and to the Logistics Management Institute. The intent was to more rapidly attain the objectives assigned in the DOD Directive. These tasks required a review of the

various management systems and procedures employed by defense contractors and the DOD to conceive, introduce and support a new system through each of its life cycle phases. Two things were obvious to the DOD at this time. One, that the scope of the problem was such that new proposals must include the close cooperation of defense contractors, thus the National Security Industrial Association was solicited as a team member, and two, that the principal objectives of integrated logistics support could not be fully attained without exploring the possible use of new scientific management tools and techniques. This latter requirement resulted in the study I will discuss with you today known as DOD Task 6.

The Air Force was assigned Task 6 in September of 1964. The task statement read "Develop a knowledge of current and proposed cost-effectiveness and similar scientific management techniques and evaluate their suitability for conducting trade off's among the elements of logistics and between the plan for use and the plan for support.

In November, I was asked to chair the joint study group organized to accomplish the task. It included representatives from the Army, Navy, Air Force, Defense Supply Agency and the National Security Industrial Association. The immediate objectives established by the group were: One, to survey the "state of the art" of techniques in Industry and Government; two, to evaluate those available for possible support of DOD Directive 4100.35; three, to disseminate the knowledge of these techniques to logistics managers and researchers in

Defense and Industry; and fourth, should time permit, to recommend additional research efforts required. It was recognized that by the 15 June 65 deadline set for study completion only a 1st step toward these lofty goals could be attained and some tasks must be identified for follow-on efforts.

The first important questions the task group faced were what is a technique and how could we identify those of value? In this regard, we felt we must be decision-oriented, that is, we must start with a specific decision performed and find that family of tools and techniques which would directly support or improve its accomplishment. Thus, the principal criteria we applied to determine if a technique existed was "that it must provide an aid in the evaluation of alternative decisions, assist in determining alternatives or improve confidence in decisions made or to be made." In all cases, the technique must be oriented to a worthwhile systems or logistics decision. Management systems, such as the maintenance management systems of the Services, were excluded as not meeting the criteria.

The "Modus Operandi" approved by the group was: First, to contact both DOD and Industry organizations likely to be developing or applying mathematical analysis techniques to defense logistics problems; two, to meet in joint sessions monthly and evaluate for usability the techniques received; three, to prepare or have prepared an abstract of each technique; and four, to develop a specialized DOD techniques bibliography and distribute it to all potential users.

Approximately 100 techniques or applications were initially received as the result of a questionnaire prepared and distributed for this purpose. This became the basis for our further inquiry and the development of the bibliographic abstracts and techniques library we are now compiling. We recognized that some organizations would not be contacted initially who may have new techniques available, but our intent has been to include these as a follow-on effort. In this regard, we have contacted only a few sources in the educational community. We have screened over 150 techniques thus far and have a current approved library of 65. Many received were duplicates or studies only and others did not meet our criteria. Approximately one-half of those received may be considered as including a cost/effectiveness trade-off.

I will now move to the principal findings and conclusions the group has made to date and, where appropriate, I will also provide you with our tentative recommendations.

1. We found that most available mathematical models are special purpose in nature in that they have been designed to solve an individual problem under a specific set of conditions. We found that they derive from a small, well-established set of generalized theories, such as queing, theory of games, etc. They have not been sufficiently generalized to be promptly applied by another user without considerable modification. Sometimes this is caused by a model having been programmed using the less commonly used machine languages, utility routines and electronic computers. The result is that generally another

user obtains the technique merely to extract the concept and theories employed and then proceeds to construct another special purpose technique.

2. As a corollary, we found that there are few generalized models available to the user. Generally, our research and development of new mathematical models has gone from the generalized theory to a specific application. It may now be appropriate to apply the case method and go from the several growing specialized models to the more generalized model capable of covering a wider range of application.

3. We found that generalized models, particularly the more sophisticated man-machine simulation models based on the mathematical and computer sciences, generally require long lead times to construct, require considerable talent to conceive and may be very costly. But, when completed, they are more effective and have more chance of survival in our present world of variability and uncertainty.

4. We found a rich potential source of available techniques to be defense and industry studies. But we also found that to identify and extract them from the verbiage will require considerable time and effort. This we plan to recommend as a follow-on task.

5. We found that many techniques were not sufficiently documented to permit an early application by another user. In some cases, only briefings are available. We are recommending additional emphasis be placed on this important phase of sharing our knowledge. Any who have experienced an application of a large

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complex problem to an electronic digital computer are aware that you seldom have time to document, but you can't afford not to.

6. Somewhat surprisingly, we found that proprietary rights was not a barrier to the free interchange of information. In fact, by using a well-documented abstract, we feel it will help consulting organizations and individuals selling their services by making known their capabilities.

7. We found that many available techniques do not provide adequate proof of validation. Adequate standards, measures and test procedures are not readily available and being considered as a necessary adjunct to the application of new techniques. This tends to continue the resistance of managers and operating personnel toward applying new methodologies. We consider this a vital missing link in the communication between the techniques innovator and the manager on whose shoulders the burden of decision rests. Our recommendations will be that we try to remove some of the aura of mystery from mathematical models for the manager and to develop more readily understood procedures for implementation.

8. We found that much effort has been spent in both defense and industry organizations toward developing "pert" type charts which isolate a series of events occurring in the life cycle of defense systems and equipment, but that little research has been conducted regarding the nature of the decisions associated with these events. One of our recommendations will center on proposing additional research in this area. Selection and use of the proper techniques from our DOD Library will be largely dependent on such a precise knowledge.

9. We found that the detailed knowledge to originate, select and apply new mathematically-oriented techniques was limited to small specialized groups in defense and industry and seldom was there any such talent in the middle-managers own functional organization. This capability was centered in operations research, industrial engineering, systems analysis and similar small research groups.

10. We considered education and training and not reorganization as the proper approach to bringing the manager and the researcher closer together. It does not appear feasible to expect a mass educational program to provide the manager with a detailed knowledge of how to construct, select and apply sophisticated mathematical models. Rather, such detailed knowledge should continue to be vested in small research and analysis groups who work as full-fledged members of the management team. But we do recommend that logisticians in key middle management positions be trained to recognize the latest types of techniques available and their limitations and capabilities in the decision-making process. Also, top management should be fully oriented to their availability and to have confidence in the application after a proper validation.

In addition, we will probably recommend the following:

1. That a single DOD agency provide bibliographic services for techniques to all users from a single Data Bank. (Logistics, Research, Production, etc.)

2. That OSD emphasize, encourage and guide development and implementation of generalized models on a broader base within each service and agency.

3. That OSD (I&L) expand the DOD logistics bibliography of techniques and to emphasize new techniques development.

4. The follow-on tasks be conducted to more fully evaluate the capabilities and limitations of the techniques received by the study group, expand the search for new techniques and maintain a high currency and responsiveness of the bibliography for the user. The time allotted for completion of the study did not permit an accomplishment of these tasks.

The Task 6 group activities have been extended to 15 July 1965 and follow-on tasks and additional proposals are now in the process of formulation.

I would like to point out that the group considers their present study only a first step toward attaining the desired DOD objectives. I hope I have portrayed to you the scope of the study and the principal findings and conclusions of the DOD task group. I should now like to close on an optimistic note - "The combination of new mathematical models to support the decision maker, coupled with the latest improvements in the computer sciences, may well provide us with a productivity for management unparallelled in our history. In the DOD, we must stay abreast of this tide."

SUMMARY AND EVALUATION OF RECENT WORK IN MEASURING THE PRODUCTIVITY OF FEDERAL AGENCIES

John W. Kendrick

Productivity indexes have long been a tool of economists for measuring changes in productive efficiency in the national economy and its various industrial branches, and their impact on economic aggregates and structure.[1] Historical measures have provided perspective for projections of aggregate production and/or input requirements and costs by industry. The measures can also be used for comparing productivity of like producing complexes at a point in time, but in this paper our focus will be on temporal comparisons.

Since World War II, increasing use has been made of productivity measurement as a management tool in the business firm.[2] Beginning in 1962, the Office of Management and Organization in the Bureau of the Budget has been encouraging Federal Government Agencies to experiment with the preparation and use of productivity measures. An initial set of pilot studies was undertaken in cooperation with representatives of 5 agencies, the results of which were reported last fall in Measuring Productivity of Federal Government Organization (Washington: U.S. Government Printing Office, 1964). Since that time, the productivity measurement group in the Bureau of the Budget has been exploring possibilities of meaningful productivity measures with approximately 10 additional agencies. As soon as staffs in those agencies are assembled (in the near future) another round of productivity studies will be launched with a formal 2-week training course for the Agency staffs, which proved successful the first time.

So far, all of the Agencies that have participated in the Budget Bureau productivity measurement program have been civilian in character. But as I describe the completed studies, you will probably see areas of possible applicability to Department of Defense programs, particularly in the field of logistics.

The Nature and Meaning of Productivity Indexes

It is important at the outset to have a clear idea of what productivity measures are and are not, and hence what they can and cannot do. Basically, productivity indexes measure the changes (or differences) in the relationship of output to input of a producing unit, in real terms; or, conversely, the changes in real costs per unit of output through time. By "real," we

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[1] For a brief history of productivity measurement, see John W. Kendrick, Productivity Trends in the United States, Chapter 1. (Princeton University Press for the National Bureau of Economic Research, 1961).

[2] See J. W. Kendrick and D. Creamer, Measuring Company Productivity: Handbook With Case Studies (National Industrial Conference Board, 1961).

mean physical units, aggregated by means of unit cost weights, held constant as of a selected base period. If outputs are related to all associated cost elements, or inputs, changes in the output-input ratio reflect primarily changes in the technology or organization of production as a result of innovations, and, over short periods, changes in rates of utilization of productive capacity and rates of adaptation to, or learning of, new ways and means of production.

By thus measuring the economics achieved through time in utilization of costly resources per unit of output, productivity indexes are one guage of management efficiency in its most characteristic function -- innovation, including the adaptation of the organization to technological changes.

Productivity indexes do not indicate whether the selection and combination of outputs are optimal with respect to the objectives of an organization -- which may be long-run profit maximization in the case of the private firm, or fulfillment of the assigned mission in the case of public agencies. Neither do productivity indexes reveal whether the inputs represent a least-cost combination, given prevailing relative input pricies and the technological possibilities. The indexes merely measure the net savings in real costs achieved in production of given outputs in successive time periods. In specifying the relevant output and input units, however, productivity measurement systems make possible unit cost estimates for the full range of relevant outputs and inputs required for optimizing decision-making.

The productivity indexes also do not measure changes in productive efficiency relative to some "norm," as in the case of work measurement. The latter systems measure actual time required per unit of work in the various operations in relation to statistically-determined, or engineered, norms. When major changes in equipment or organization are made, the norms are changed, so that the measures are discontinuous. In contrast, productivity systems attempt to measure the units of final output (not all types of work units) of the organization continuously through time adjusting where necessary and feasible for quality change; the weights are not "norms" but merely unit costs of a base-period; by comparing aggregate output with similarly weighted inputs, the results of the cost-reducing technological changes are revealed.

Finally, while productivity measures may concentrate on labor time alone, in order to show changes in unit manhour usage (or its inverse, "output per manhour"), more comprehensive measures relating to all inputs, or cost elements, are more desirable. The total productivity ratios make possible the calculation of the effects of factor substitutions on individual input requirements, as well as showing the net economies in real costs achieved over time.

Components of the Productivity Measures

To really understand the significance and potential uses of productivity measures, it is necessary to understand just how the output and input components of the measures are defined and constructed. We shall describe each of these first in general terms, then illustrate concretely from the Budget Bureau case studies.

Output

To be susceptible to productivity measurement, an organization must produce relatively standard types of outputs from one period to the next. Numbers of units produced must be counted in terms of the smallest, homogeneous type of commodity or service rendered. Popular opinion to the contrary, it is as easy to define and measure types of services as it is commodity-types. In the case of General Government, however, the alternative to weighting physical units -- the "deflation" of production values to correct for price changes -- is not available since Government services are not priced and sold on markets as is private industry output. This restriction does not generally apply to inputs, however. Fortunately, the development of work measurement systems and other performance measures has proceeded far enough in most Federal agencies to provide most or all of the required physical volume of service output data for productivity measurement. Cost accounting systems generally provide the data required to weighting units of the various types of outputs (and inputs).

When, as frequently occurs in a dynamic economy from time-to-time, the "quality" or resource content of the output changes, an adjustment to the succeeding output units should be made to preserve comparability with the older model products.

It is also important that only final products be measured. These are the goods or services which implement the basic mission of the agency in servicing the public or other agencies. Staff services, such as personnel administration, custodial and internal maintenance services, which are "intermediate" and merely facilitate performance of basic missions, should be excluded. In this respect, the scope of productivity and work measurement systems differ. Increasing efficiencies in intermediate production indirectly affect productivity indexes; they could be shown explicitly though the ratio of intermediate to final work units based on work measurement data.

The more consolidated government organization productivity indexes are made, the more outputs are excluded as intermediate. In the Department of Defense, it would be very difficult to define and measure the final services of the entire Department to the public. It is much easier to define and measure various logistical and other support services; which suggests that DOD organizational productivity measures would better remain unconsolidated unless the problem of measuring final defense services is solved.

Although intermediate outputs are excluded, the production of investment outputs should be included. These outputs, which may be consumed within the organization but only over a number of accounting periods, themselves render input services over their lifetimes.

So much for general observations. Let us now see how the several agencies included in the Budget Bureau pilot studies defined their final outputs.

Division of Disbursement -- The simplest set of outputs of all those studied was provided by the Division of Disbursement in the Treasury Department. Only two separate outputs were defined: the number of payments made on behalf of other Government agencies, and the number of savings bonds issued to payroll plan savings bond subscribers. The payments were of various types, such as payments of veterans benefits and NSLI dividends, social security benefits, tax refunds, and salary payments by the various agencies -- but no differentiation was made since the payment unit was relatively homogeneous regardless of object.

Very significant subsidiary data were available, however, showing numbers of payments by method (see Table 1). Since the corresponding input data were also available, it was possible to compute productivity by level of technology.

TABLE 1

Division of Disbursement: Number of Savings Bonds Issued, Cash Payments, and Checks Issued by Method of Preparation (Selected years; in thousands)

	Fiscal Years		
	1950	1956	1962
Savings bonds issued	2,486	2,854	3,999
Cash payments	664	0	0
Checks issued (by method)			
Typed	51,204	15,900	12,984
Addressing machine	129,921	163,700	8,837
Manual transfer ported	13,823	8,171	0
Bill feed	0	14,249	107,199
Automatic transfer ported	0	2,094	0
Thermal printed	0	19,560	3,029
Semi-electronic	0	0	67,941
Stencil	0	0	3,552
EDP	0	0	114,120
TOTAL	198,098	226,528	321,661

Source: Measuring Productivity of Federal Government Organizations, Table 21, p. 108. Annual data are shown in .he original table.

Department of Insurance, VA. -- The Insurance Service of the Veterans Administration is engaged in providing life insurance coverage with all of the services provided by law. The services were defined as consisting of 5 main types: (1) the basic service is provision of insurance protection, as reflected in the average number of policies in force during successive periods; (2) a distinct service is the waiver of premium payments in cases where the insured become totally disabled, as measured by the number of initial claims and the number of review decisions; (3) termination service consists of making final settlements, but costs differ according to the several ways the insurance policy is terminated; (4) the number of loan applications measures the service of lending to policyholders against accumulated equity; (5) a fifth category consisted of the numbers of new policies issued (two types). The volume of the several types of output for selected years is shown in Table 2.

TABLE 2

VA Department of Insurance: Volume of Outputs, by Type
(Selected years; in thousands)

	1955	1959	1962
Average number of policies in force	6,469	6,439	6,052
New policies issued ("service disabled" and "other")	212	6	4
Selected terminations:			
Death	20	23	27
Lapse	335	63	91
Cash surrended and matured endowment	30	15	15
Disability claims:			
New claims	29	18	20
Review decisions	65	53	58
Loan applications	97	106	116

Source: Measuring Productivity of Federal Government Organizations, Table IV-1-1, p. 183.

Although the Department of Insurance computed an output index based on all 9 major types of output, it relied principally on a 5-output index for reasons of simplicity. Both types of policy issue were of negligible importance in recent years, and were omitted; and only one lapse series, instead of three, was used.

Post Office Department -- The final outputs of the Post Office Department were defined in terms of 14 classes of mail, and 7 types of services (see Table 3). The multiple physical volume measures shown for second- and fourth-class mail improve the homogeneity of the weighted aggregates without involving duplication since the weights for these classes of mail are split among the measures of the several dimensions of output. The measures shown in the table cover about 98 percent of total Post Office product. Box rentals and various non-postal services, such as alien address reporting, were omitted.

An interesting aspect of the Post Office study was the derivation of component output indexes for the following broad areas of total manpower: (A) mailhandling, window service and related maintenance; (B) collection and delivery service, and related maintenance; and (C) administration and related field services. The same outputs were used, but weights were split by component manpower usage.

Weighted output indexes (based on allocations of total costs) were also computed for the following segments: (I) personnel costs; (II) purchased nonlocal transportation service; and (III) building occupancy (rent). These segmental and associated productivity measures are helpful in quantifying the contributions of the various parts of the Department to productivity change.

Systems Maintenance Service, FAA.-- The product of the Systems Maintenance Service of the Federal Aviation Agency is defined as "a facility in operation." The measure involves both the number of facilities maintained, by type, and the period of time over which they are maintained in operational condition. The various types of facilities are weighted by their base-period (FY 1962) labor maintenance costs. If the weighted aggregate is divided by the average annual labor cost of 1962, a common denominator of output, the "Standard Facility Year" is derived.

The various major types of facilities, and the associated Standard Facility Years, are shown for selected years in Table 4. Actually, Group I, Air Navigation Outputs, includes outputs for 23 different facility types, subdivided into 56 classes and subclasses. Group II, Air Traffic Control Outputs, includes outputs of 44 facility types, subdivided into 211 classes and subclasses; and Group III, Nonaeronautical Outputs, include outputs of 16 facility types, subdivided into 51 classes and subclasses. Thus, the three Groups together comprise 83 facility types, and 318 classes and subclasses.

An interesting feature of the Systems Maintenance Service study was the preparation of separate output and productivity indexes for the 6 FAA regions. A useful quality indicator, facility outages, was prepared by cause of outage time, by region.

TABLE 3

Post Office Department, Quantities of Output by Class Of Mail and Service, FY 1953, 1958, and 1962 (In millions)

Class of mail and service	Measure	1962	1958	1953
1st-class mail	Pieces	35,332	32,218	27,257
Domestic airmail, letters and cards.	do	1,513	1,410	1,412
Domestic airmail, parcels	do	32	24	18
2d-class mail	Pieces	8,090	7,148	6,762
	Pounds	2,908	2,695	2,497
	Cubic feet	133	127	115
Controlled circulation publications.	Pieces	165	145	56
3d-class mail	do	17,837	16,064	12,210
4th-class mail--"books" (educational and library materials)	Pieces	153	93	68
	Pounds	636	446	300
	Cubic feet	35	23	15
Other 4th-class mail	Pieces	871	862	970
	Pounds	4,938	5,254	6,197
	Cubic feet	508	534	541
International surface mail	Pieces	287	353	336
International airmail, letters and cards	do	218	181	148
International airmail, parcels	do	1.3	0.98	0.86
Penalty mail	do	1,877	1,560	1,658
Franked mail	do	111	67	49
Free-for-blind mail	do	6	4.1	2.7
Registry service	Items	55	57	90
Certified mail service	do	31	24	0
Insurance service	do	165	158	198
Collect-on-delivery service	do	24	29	49
Special delivery service	do	92	96	114
Money order service	Money orders	252	312	370
Postal savings service	Certificates	2.7	6.1	20

Source: Measuring Productivity of Federal Government Organizations, Table 45, pp. 208-209. Appendix VI-1 for a complete listing of all types and classes of facilities.

TABLE 4

SYSTEMS MAINTENANCE SERVICE, GROWTH OF OUTPUT
by Major Type of Facilities, 1958-62

	Standard facility years		
	FY 1958	FY 1962	Increase
Air navigation:			Percent
VHF omnidirectional facilities	403.12	610.53	51.5
TACAN and distance measuring equipment	123.31	592.49	380.5
Low-frequency ranges and fan markers	294.71	262.26	-11.0
Homing beacons	59.82	68.72	14.9
Instrument landing systems	285.06	374.67	31.4
Approach light systems	43.52	187.32	330.4
Beacons and fields	120.83	60.50	-49.9
Group I -- subtotal	1,330.37	2,156.49	62.1
Air Traffic control:			
Towers, stations, and centers	1,709.27	3,888.62	127.5
Radar systems	239.23	884.37	269.7
VHF and microwave links	113.59	481.35	323.8
Direction finders	31.26	49.55	58.5
Teletypewriter systems	126.01	206.81	64.1
Engine generating equipments	52.34	46.87	-10.5
Group II -- subtotal	2,271.70	5,557.57	144.6
Nonaeronautical:			
Housing, utilities, and miscellaneous	318.30	386.21	21.3
Group III -- subtotal	318.30	386.21	21.3
Total	3,920.37	8,100.27	106.6

Source: Measuring Productivity of Federal Government Organizations, Table 58, p. 264, See Appendix VI-1 for a complete listing of all types and classes of facilities.

Bureau of Land Management. -- The mission of the Bureau of Land Management in the Department of the Interior is, generally "...to manage the Federal lands under its jurisdiction according to the best conservation practices and to make available the land and resources in a manner consistent with the public interest as provided by law."[3] The outputs of the Bureau proved difficult to define in operational terms, in part because its end products consist of "investment" outputs representing improvements in resources as well as of "current" outputs consisting of direct services to individuals and organizations. In part, the difficulties were associated with a variable quality content of the outputs as measured. Due to these difficulties, and lack of certain strategic data, the Bureau was not able in the time available to measure investment outputs, and the current output measures which were developed for 3 of the 5 programs were considered to be interim in nature. As a result, no composite agency measure was possible and no Bureau productivity measure is included in summary Table 6 below.

The five major programs of the Bureau are shown in Table 5. The manhours allocated to current outputs, as distinct from investment outputs, are based on programming data. No output measures were developed for the Range and Recreation programs, which absorbed a bit under 20 percent of manhours devoted to current output in FY 1962. For the Lands and Minerals program, measures were devised based on number of cases closed in response to applications by the public for leases, grants and sales in the categories listed in the table. An alternative measure was based on positive decisions (issuances and allowances). Since the numbers of denials rose sharply relative to positive decisions over the period 1952-62, the alternative measures show quite different results. A weighted aggregate of the several types of decisions would seem to have been a reasonable compromise.

In the Forestry program, the measure was based on the number of broad feet of timber cut and the amount offered for sale. A decline in this composite index between 1952 and 1962 was apparently due to increased manpower requirements for constructing access routes, for classifying timber resources, for preparing timber tracts for sales, supervising cutting, and for administering sales contracts. This suggests that not all manhours associated with investment were eliminated from the current programs, as well as indicating the need for a quality adjustment factor to current output.

In Cadastral Surveys, output was measured by the number of miles of line surveyed and the number of monuments set for users outside the Bureau. Sharp fluctuation in this index suggest that more research is needed to allow for variations in local conditions and other qualitative factors before the index is finalized.

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[3] Ibid., p. 301.

TABLE 5

Bureau of Land Management, Estimated Distribution of Total[1] Manhours Worked, by Program and Type of Output, FY 1962

Program and type of output	Man-hours allocated to current outputs	Man-hours allocated to investment outputs
Lands and minerals program total	1,803,541	681,731
Agricultural cases	163,085	n.a.
Exchanges	195,702	n.a.
Sales of land	271,808	n.a.
Selections	184,830	n.a.
Land title cases	163,085	n.a.
Leases and permits (land)	108,723	n.a.
Mineral entries	179,077	n.a.
Oil and gas leases (on land)	358,155	n.a.
Oil and gas leases (OCS[2])	71,631	n.a.
Material sales	35,815	n.a.
Leases, permits, licenses (mineral)	71,631	n.a.
Forestry program total	664,589	791,861
Timber offered for sale	531,671	n.a.
Timber cut	132,918	n.a.
Cadastral surveys for other agencies total	77,036	0
Miles of survey line	30,814	0
Monuments set	46,222	0
Range program total	557,897	1,156,053
Recreation program total	1,548	6,194
BLM total	3,104,612	2,635,539

n.a. = Not available.

1 Excludes 10,036 man-hours devoted to fighting of fires

2 Outer Continental Shelf.

Source: Measuring Productivity of Federal Government Organizations, Table 65 p. 326.

TABLE 6

ANNUAL GAINS IN PRODUCTIVITY OF FOUR FEDERAL GOVERNMENT ORGANIZATIONS
(average percent per year)

Organization and period	Output per man-hour	Output per constant dollar of payroll cost	Output per constant dollar of total cost
	Average*	Average*	Average*
Department of Insurance Veterans Administration 1955-62	9.8	8.3	[1]6.9
Division of Disbursement, Department of the Treasury, 1949-62	[2]9.4	8.6	[3]6.1
Post Office Department, 1953-62	0.3	0.2	[4]0.4
Systems Maintenance Service, Federal Aviation Agency, 1958-62	-4.1	.1	n.a.

*Period averages were calculated by the compound interest formula (annually compounded) applied to the values of the productivity index in the first and the last year of the period.

n.a. = Not available.

[1] Total budget cost.

[2] Output per man-year.

[3] Excludes postage.

[4] Includes personal services, transportation, and space occupancy costs.

Source: Measuring Productivity of Federal Government Organizations, Table 1, p. 14.

Despite the lack of definitive results, apparently the interim results have been of some use to Bureau management "... in that the process of output identification contributed to clarification of goals and objectives and outlined needs for additional information."[4]

Inputs

The measure of resource inputs, or cost elements, used in an organization should have the same scope as the output measure. Further, as we noted earlier, it is desirable if all inputs are measured so that net savings in total real budgeted costs per unit of output can be calculated. All of the agencies prepared measures of the predominant labor input, and 3 of them developed measures that purported to cover all, or most, of the budgeted costs.

The first measure of labor input was based on employee time. Manhours worked were calculated in the Department of Insurance, the Post Office, and the Systems Maintenance Service. In the other two agencies, full-time equivalent man-years worked were calculated, although in the Bureau of Land Management man-years were converted into estimated manhours by a uniform conversion formula.

A more refined variant of the labor input measure was computed in four of the agencies (other than BLM). Data on manhours or man-years worked were assembled by grade (by occupational category in the Post Office case) and then weighted by base-period average labor compensation in each category. This technique in effect adjusts for different qualities of labor. If there were a relative shift of labor to higher-paid grades or occupations, the "real labor cost" index would rise relative to the labor-time index. This was generally the case, except for the Systems Maintenance Service in which there was an average downgrading of labor after 1958.

Of the 3 agencies with more comprehensive real cost measures, the VA Department of insurance included total budgeted costs; the Division of Disbursement included all budgeted costs except postage; while in the Post Office Department transportation and building occupancy costs were included in addition to payrolls. The nonlabor costs, including equipment rentals, were generally "deflated" by more or less appropriate price indexes.

No allowance was made for an implicit interest cost and depreciation on capital goods commanded by the agencies. This would seem to be a defect in the total real cost measures, associated in part with deficiencies in Federal Government cost accounting procedures. Neither was a real cost estimated for the floor space and utilities furnished free of charge by the General Services Administration. Nor were real charges computed for a share of overhead of parent agencies, or outside service agencies such as the Civil Service Commission.

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[4] Ibid., p. 304.

While there are advantages in sticking to real budgeted costs, more comprehensive analysis of productivity change would require computation of alternative indexes based on all inputs utilized, whether explicitly charged to the agency or not.

Weighting

Fiscal year 1962 was chosen as the base-period for output and input weights. Theoretically, relative weights can affect the movement of output or input aggregates and thus of productivity ratios. Weighting experiements with the Post Office measures indicated that the selection of the base-period from the available post-War years made relatively little difference. Nevertheless, consistency of base-period (and other aspects of methodology) among the agencies is a desirable feature of Budget Bureau leadership in the productivity studies.[5]

The principle of symmetry between the types of weights used to combine outputs and the range of inputs with which output was compared was another desirable feature of the agency measures. In the Division of Disbursement, for example, where man-years, constant dollar payrolls, and total real costs were each related to output, the individual outputs were combined by the three corresponding sets of weights. By this procedure, the overall productivity index is an internal mean of its components, and remains unaffected by relative shifts of resources from one type of activity to another within the organization.

Results of the Studies

The average annual rates of change over the periods covered in the several types of productivity measures for the Federal Government organizations studied are summarized in Table 6. There was considerable variability in the annual percent changes, as is also true of private company and industry measures.

The high rate of productivity advance in two of the organizations, compared with the rate of change in the other two, is striking. The high rates compare favorably with productivity advance in the most dynamic private industries over comparable time-spans. It should be noted that the dispersion of rates of change narrows as output is related to progressively broader input measures. This is due to the fact that the very high rates of gain in output per manhour were purchased in part at the cost of considerable up-grading of labor, and significant increases in captial and other inputs per employee.

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[5] Weighting and other aspects of index number methodology are discussed in detail by Bruce D. Mudget, Index Numbers (John Wiley and Sons, 1951). See also the classical work by Wesley C. Mitchell, The Making and Using of Index Numbers, Bulletin 656, Bureau of Labor Statistics, U.S. Department of Labor.

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The Budget Bureau observes due caution in appending the following note to its summary Table: "The data shown in this table do not warrant comparisons of management effectiveness in individual organizations because these estimates of actual productivity gains realized do not make any allowance for possible differences in the potential for productivity improvements that existed during the period." Yet one wonders if there is not considerable room for improvement of productivity performance in the two low agencies. Possible management uses of the productivity indexes as a tool for spurring productivity advance and other purposes are discussed in the following section.

The experience of four markedly different organizations is admittedly a narrow basis for generalization. Yet the report points out that the organizations with rapid productivity advance were characterized by (1) radical changes in production technology; (2) uniform (i.e., not diversified) output; and (3) a comparatively small number of locations. In constrast to experience in private industry, there was not a positive correlation between relative increases in productivity and in output, possibly due to the irrelevance in government of the tendency for relative productivity increases to be associated with relative price declines and relative sales advance.

As the number of cases in the budget bureau file grow, so will possibilities of generalization regarding causes of increasing productive efficiency. This type of analysis will also be promoted by detailed agency descriptions and analyses of their own historical records. In the Treasury Department case, for example, not only are the technological developments detailed, but an interesting attempt is made to estimate the dollar savings resulting from the major types of managment improvements.[6] Broad dissemination by the Office of Management and Organization of information concerning successful cost reducing innovations in given agencies to all agencies could be significant force in accelerating the diffusion of innovation.

Evaluation

Methodology

Technically, I believe the Federal agency productivity measures are constructed according to sound principles (with the possible exception of the treatment of capital inputs), and compare favorably with the best of those available in the private sector of the economy. Once the basic quantity measures are done, however, more effort should be devoted to devising appropriate measures of quality change, such as the out-

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[6]Ibid., Appendix III-1, p. 124, It should also be noted that between 1948 and 1964, approximately half of the regional offices of the Division of Disbursement had been closed(see Appendix III-2, p. 125.)

age index developed by the Systems Maintenance Service. Quality measures are important in their own right, as well as aiding the interpretation of productivity indexes. I have been informed that the Budget Bureau plans to put more stress on quality measures in their second round of productivity studies.

It should also be pointed out that once estimation methods have been worked out, the Agency cannot then proceed to continue computing the productivity indexes in a mechanical way. Continuing "repair and maintenance" is required, particularly to identify modifications of outputs, new services introduced and old ones dropped. Consideration should be given periodically to possible changes in output and input weights, when relative unit costs have changed significantly. Whenever possible, improvements to the measures should be made when new data (as on non-labor inputs) become available or are developed specially for the productivity measures.

Management Use

The degree of usefulness of productivity indexes to the management officials of Government agencies remains to be proved. As summary measures, productivity indexes are of primary interest to top management in agencies and their organizational divisions. At best, they merely serve to signal the need for investigation, and the asking of further questions. A sluggish, or declining, overall index is, of course, cause for general concern. The components of the index, disaggregated by organizational units, by types of service, and by types of input, will help to pinpoint a typical experience. In some cases, the seemingly unfavorable behavior of an index may be quite explainable, as when a partial productivity ratio drops as a result of substitution of the measured input for another input. In other cases, unfavorable index changes may lead to investigations that uncover inefficiency.

Particular caution must be employed in interpreting differences in productivity levels for various installations performing like functions. In some cases, differences in productivity and in particular input requirements may be entirely due to differences in size of the installltions, locational factors, and the like. In other cases, the differences may be due to genuine differences in management efficiency in the several installations.

Obviously, the usefulness of productivity measures will depend ultimately on management's desire to use them. Further, the more detailed the measures, the more potentially useful. Quarterly measures are probably more useful than annual measures. On the other hand, monthly measures will tend to be influenced by erratic factors and may not be significant indicators of trends.

The cliche is often heard that efficiency measures, such as productivity indexes, by their very existence tend to spur the drive for greater management efficiency. This is possibly true, but with the current proliferation of various measures, an additional measure must win a place on its own merit. My guess is that, as a summary measure of new changes in productive efficiency and in unit requirements for the several inputs, productivity indexes will win a secure place in the battery of management statistical tools. But it is as an aid in budgeting and long-range projections that I believe the more important uses of productivity measures lie.

Budgeting and projections

The logic behind the use of productivity measures for budgeting is simple. After the workloads of the agency are projected, projections of the total and partial productivity ratios yield the input requirements consistent with the workload and productivity projections. The projected input requirements, when combined with input price projections, yield the cost totals that must be budgeted for. Essentially the same technique can be used in 5-year or longer projections.

In the case of budgets, however, the productivity projection can in part be based on the expected effects of planned technological changes. This is particularly feasible when the unit input requirements associated with the new as compared with the older technologies are known, as well as the planned rate of investment in new equipment, as is the case in the Division of Disbursement.[7] The further into the future productivity projections are pushed, the greater must be the reliance on extrapolation of past trends. For this reason, the accumulation of considerable historical data and estimates of productivity change is desirable.

The same principle holds with respect to input prices. Short-term forecasts can take into account known supply and demand forces, such as labor contracts. Longer-term projections can take into account past relative price trends, but fewer specific market forces as the projection horizon is extended.

The short-term projections of relative price movements are important in estimating the least-cost combination of inputs, which together with the projections of technological possibilities, should affect the unit input requirement projections. The longer-term trends in relative input prices in conjunction with the productivity projection, should also affect the input projections in at least a general way.

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[7]The projected productivity change might also be announced as a "goal," if this would be deemed to have an incentive effect.

The possibilities in the use of projectivity data in budgeting and projections have been recognized by Office of Budget Review in the Bureau of the Budget, and by budget officers in the agencies. Certainly, the decomposition of dollar data into price and quantity elements is an important statistical and analytical step forward.

Directions for Future Work

It is to be hoped that the Office of Management and Organization in the Budget Bureau will continue to provide leadership in the field of productivity measurement and analysis. Centralized leadership helps to provide standardization of measurement methodology which conduces to inter-agency comparisons, and promotes the sharing of fruitful technological innovations across agency lines.

Productivity studies should be encouraged in all organizations most of whose outputs are sufficiently homogeneous through time to permit meaningful measures. It is my impression that most civilian agencies would fit this description, and some of the functions of the Department of Defense particularly in the supply and maintenance areas. Certainly, the possibilities should be thoroughly explored.

Once further progress has been made in Federal Government productivity studies, comparisons might usefully be made in some areas with private industry; and in many areas with counterpart functions in Governement of other countries.

The studies will be futile, however, unless they are used to promote technological and organizational advances in Government, including improved budgeting procedures.

SYSTEMS RESEARCH MEMORANDUM No. 123

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CHANCE-CONSTRAINED PROGRAMMING AND RELATED APPROACHES TO COST EFFECTIVENESS

by

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SYSTEMS RESEARCH GROUP

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CHANCE-CONSTRAINED PROGRAMMING AND RELATED APPROACHES TO COST EFFECTIVENESS

1. <u>Introduction</u>:

This paper is intended to supply a broad survey of recent research for its possible bearing on cost-effectiveness measures and related aspects of planning and evaluation. At the outset, then, we might well identify some aspects of the points of view which have governed this survey. <u>Inter alia</u>, this will also help us to present an overview of the structure and the course of the developments pursued in the sections that follow.

First, we believe that such a survey may best be conducted by according the term "cost/effectiveness" only generic significance so that it then covers any system for measuring benefits and the resources that must be expended to secure them. Second, we believe that any meaningful system of measures must be related to an underlying model and that, further, it is generally advisable to make these models explicit whenever the resulting measures are to be used for analyzing complex systems. Third, the models we deal with are of a so-called optimization variety and, in particular, the resulting cost-effectiveness measures and related trade-off evaluations depend on these optimality properties for their full significance. Fourth, the models and the related research we examine are of a so-called constrained optimization variety. Fifth, we are concerned to note that such constrained optimizations are not confined either (a) to those types of problems where only a single measure of a "best" result can be defined[1/]

1/ See, e.g., pp. 379 ff. in C. J. Hitch and R. N. McKean [37] for discussions that are pertinent to "efficiency" in a military context. (Numbers in square bracket refer to the references listed in the bibliography at the end of this paper.)

or (b) to those situations in which the numbers (and related symbols) must have all of the properties usually accorded to them in ordinary arithmetic.[1/] Finally, we should also note that the formulation of such a model is a beginning rather than the end of a process that extends from using the model as a mode of clarifying objectives and extends through its use as a guide to data collection and, ultimately, reformulation of the model itself.[2/]

With these points in mind we now summarize the remaining portions of this paper. In section 2 a _précis_ is supplied of the processes for effecting cost-and-benefit calculations which are standard in economics and accountancy. Features of the underlying models are identified by reference to the optimality assumptions which often, perhaps implicitly, accompany such analyses. Section 2 then concludes with a delineation of extensions that might well be effected to other contexts--as is now possible--by using the results of research (and accompanying applications) in linear programming and its subsequent extensions.

In section 3 we proceed to supply a variety of examples, with special reference to chance constrained programming, as one way to lend concrete significance to some of our preceding characterizations.

The models (i.e., those which have been customarily employed for many years by industry and government) described in section 2 often proceed in so-called _ceteris paribus_ fashion--one variable is altered while all others are held constant[3/] and so, by way of contrast, an

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1/ See, e.g., R. J. Aumann, "Subjective Programming" in [60.1] and see also pp. 280-282 and pp. 753-757 in [12].

2/ _Vide_ [10] and [16].

3/ See, e.g., Hitch and McKean, _loc. cit._ [37] p. 362.

alternate approach is next illustrated in section 4 by reference to a linear programming example for ship-assignments to accomplish certain missions. This ship-to-mission assignment model is restricted only to a deterministic context so that particular attention might thereby be directed in a fairly simple way to the so-called dual evaluators--which proceed in _mutatis mutandis_ fashion to examine the consequences of altering requirements by supposing that all relevant variables are optimally adjusted to each such alteration.[1/]

As we have already indicated, the model used in section 4 is both simple and deterministic. It is, in fact, a linear programming model. But this is not the only way in which we shall treat this model. Recent years have witnessed research that has been directed to expanding ordinary linear programming so that constrained optimizations may be employed in contexts where probabilistic considerations must be explicitly accommodated. This is illustrated in the sections that follow. For instance, in section 5 the topic of "stochastic linear programming" is illustrated by indicating how an entire probability distribution of optimal "figures of merit" may be derived when random elements are present. But this, it should be noted, is only one of the possible approaches that have emanated from the research that has now been undertaken in these "probabilistic programming areas." Note, for example, that stochastic linear programming assumes that all constraints are always satisfied.[2/] In some situations, however, it may

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1/ See pp. 16 ff. in [12] for further discussion of such _ceteris paribus_ and _mutatis mutandis_ adjustments.

2/ This is also true of "linear programming under uncertainty," as formulated by G. B. Dantzig. See, e.g., [29] and [30]. For reports of other recent research on linear programming under uncertainty see [22] and [23]

be necessary to consider the possibility of constraint violations in a way that enables one to prescribe, control and evaluate the attendant risks for each component (constraint) in a complex system.

This brings us to the topic of "chance constrained programming" which occupies Sections 6 and 7. The former introduces this topic in a very simple context 1/ and concludes with suggestions for ways in which new measures may be derived from an already attained solution. The latter introduces a variety of objectives 2/ and discusses how these may be combined with other considerations to produce both different and similar results.

In section 8 we again return to deterministic situations in order to obtain entry into the topic of multiple objectives. This is done by reference to the ideas of "efficiency" as developed by T. C. Koopmans 3/ which are then reduced and simplified by recourse to already available artifacts. 4/ Finally, in section 9, the idea of multiple objectives is extended 5/ and adjusted to problems stated explicitly in probabilistic form. The findings and evaluations that are stated at various points

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1/ I.e., by reference to the so-called zero order rules of chance-constrained programming

2/ With the statistical characterizations that are generally required for the so-called higher order rules of chance-constrained programming.

3/ See [42].

4/ As presented in Chapter IX of [12].

5/ By reference to the ideas of "functional efficiency" as presented in Chapter IX of [12].

throughout the manuscript are then extended in section 10 for their bearing on research issues and possibilities for practical implementation with special reference to the topics covered in section 9.

2. One-Dimensional Approaches to Cost and Benefit Measurements

The measurement procedures for effecting choices in private enterprises might be idealized along lines like the following. First, all pertinent attributes are identified in their respective dimensions. Second, only those attributes which are variable for the decision under consideration are singled out for explicit attention.[1/] A single measure of "value" is then assigned to each such variable in a way that makes it possible to obtain a unidimensional scale along which comparisons can be effected for ascertaining the "costs" and "benefits" accruing from each possible state that might be assigned to the underlying attributes.

This is a very general description of procedures that are supposedly employed for effecting choices in many contexts. Thus in the literature of economics and cost (and managerial) accounting, for instance, this kind of reduction is supposed to occur in preparation for reaching a best decision.[2/] In the cost accounting literature, in any event, the relevant scale is a monetary one. Along the indicated scale a best decision will then be one in which produces results like the following:

1/ I.e., those attributes which can be made to vary in some way--perhaps discontinuously--and thereby also produce a variation along the indicated scale.

2/ See, e.g., Chapter 4, "A Comprehensive Restatement of the Theory of Cost and Production" in P. A. Samuelson, [56] as well as pp. 47- 50 ff. in R. Beyer [7].

(a) net benefits are adequate to cover all of the indicated variable costs and (b) no other choice will produce a greater positive excess in the difference between benefits and costs. In particular, then, a best choice will be one which cannot be bettered by increasing or decreasing the value of any variable. That is, no variable, in its own dimension, can be altered to produce a better result as measured on the indicated one-dimensional scale.

Various further refinements would be needed to complete this description. Relevant extensions would include considerations of stability, variations over differing intervals of time, related issues of risk or uncertainty and possibly other desiderata as well. These issues have already been well attended to by others, however, and since the results of this prior research are now readily available we need not dwell upon them here.[1/] Our objective is rather to examine some further extensions from more recent research and so, for background only, we now summarize some of the salient features and assumptions of these prior approaches:

A. Assumptions of Optimum Choice and Technology: It is usually assumed that all adjustments are optimally effected. By this we mean that (a) all costs are held to a minimum at each possible value of the related variables and (b) all benefits are similarly pushed to their corresponding maximum values. In other words, relative to the indicated one-dimensional scale, the cost and revenue (benefit) functions are each supposedly at the minimum and maximum values admitted for each of the variations being considered. Given this assumption of a "best" set of functions, the decision problem then reduces to choosing values for the relevant variable in order to maximize the difference in the values that will be secured, as scored from these already erected

1/ See the references already cited as well as F. and V. Lutz, [45] and S. Alexander, et. al. as cited on p. 147 of [2].

benefit and penalty functions. Thus, for terminological convenience we may divide this into a two-stage process: At stage one, a best set of functions is determined. At "stage-two" the relevant variables are then assigned preferred values by reference to their cost and benefit consequences. We should note that the stage one and stage two choices are generally effected by reference to the same one-dimensional scale so that, in particular, the choice of such a scale must presumably be made prior to the stage-one choices. We should also note that the stage two variables which are relevant, as well as their range of possible variations, etc., is decided (at least in part) at stage one.

A.1 <u>Discussion and Illustration:</u> In commercial and governmental practice, such stage-one decisions may be viewed as revolving around choices of equipment, plant layout and design, and even a "best" organization arrangement. 1/ Given such stage-one determinations, the stage-two decisions may then be collectively interpreted as being confined to choices of input-output combinations which best meet an organization's requirements and opportunities. Both sets of decisions are effected, in any event, by reference to their consequences as these might be judged and evaluated along some pertinent one-dimensional scale of values.

A.2 <u>Remark</u>: To some extent this stage-one and stage-two division is arbitrary. For instance an intermediate category may also be introduced, when necessary, to allow for the development of operating guides--e.g., standard costs and budgetary relations, as well as bills of materials, manning tables and planning factors. Time-and-motion and methods studies--and related managerial activities--may be regarded as members of this intermediate class. We should then note that such studies often include allowances for training workers and effecting equipment changes. Thus, as these examples illustrate, this intermediate category may alter the previous stage-one decisions and enter into the subsequent stage-two decisions as well. In any case, resource expenditures must then also be considered relative to such intermediate activities and these, too, must be calibrated and evaluated along the one-dimensional scales which

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1/ A more precise characterization would distinguish elements within each of these categories that continue to be variable at stage two. See, e.g., Sune Carlson [8] pp. 75 ff. and F. Levy [43] *passim*.

we are presently considering. 1/ By means of suitable mathematical expressions, we shall later try to make these statements more precise. For the moment, however, we may note that the need for distinguishing such categories is dependent, in part, on the available technologies from which choices must be made. It is also dependent on the managerial tools and hence on the results that research can make available for effecting simultaneous stage-one and stage-two--as well as intermediate-stage--choices in realistic contexts.

B. Data Requirements and Availability: Assumptions of data availability and use are also moot. Such assumptions evidently apply when a choice of scales is being considered. They also cover the initial task of identifying and classifying the relevant variables as well as the further tasks of determining the relations that govern the behavior of these variables insofar as these bear on the values that will subsequently emerge on any value scale that will subsequently be employed. These remarks are intended to be sufficiently general so that they will cover managerial decisions in either probabilistic or deterministic contexts. 2/

B.1 Deterministic Analysis and Decisions: It is usually assumed that all pertinent data are available as needed at relatively trivial expenditures of cost and effort. This assumption, however, needs to be interpreted rather carefully. Thus, for instance, as we have already emphasized, costs (and benefits) which are fixed may be ignored. Furthermore, when a marginalist or incremental approach is being used one may suppose that only a knowledge of the increments to cost and benefits will provide all of the requisite information. Indeed, it can be shown--in a very general class of cases--that even the increments to cost need not be available provided that other, perhaps more readily obtainable, "surrogate" data are at hand. 3/

B.2 Probabilistic Analysis and Decisions: Questions of data availability are also pertinent here, of course,

1/ See, e.g., [17] for a detailed development of models and related modes of analysis and computation.

2/ In some cases the latter might be further subdivided into situations of "risk" (probabilities are known) and of "uncertainty" (probabilities are not known).

3/ See the discussions and further references supplied in Chapter XI of [12].

and unless so-called "certainty equivalent" 1/ or "deterministic equivalent" 2/ reductions can be effected one may suppose that the data requirements will generally be more onerous than in deterministic cases. For instance, the solution of a probabilistic problem may make it necessary to determine parameter values or even entire probability distributions. This may be a difficult job. 3/ It may even be impossible without recourse to research that will extend the boundaries of existing statistical theory, perhaps, or at least increase the efficiency of some of its procedures for estimation and computation. Even this may not be wholly adequate, however, since still deeper issues may arise concerning the choice of a "rational" objective which, in turn, can also raise many questions concerning the choice of suitable scales for measuring and evaluating the consequences of possible choices for decision.

B.3 Remark: We shall return to this latter topic in subsequent portions of this paper. Here we should note that one course of argument runs to a development of "utility functions" in order to produce yet another one-dimensional scale which will, supposedly, resolve the choice-of-objectives problem noted under B.2. There may be practical difficulties in the way of such an approach, however, and these may be compounded--even as a matter of principle--when more than one decision maker is

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1/ See, e.g., Chapter 6 in [38].

2/ This terminology is borrowed from the literature of "chance-constrained programming."--See, e.g., [11].

3/ Some relief may be secured if one is willing to proceed via a judgmental (or subjective) assignment of the relevant probabilities. See, e.g., H. Raiffa and R. Schlaifer [54] or, at a more elementary level, see R. Schlaifer [58].

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involved. 1/ Furthermore, one may observe that there is no *a priori* reason to suppose that the resulting "utility scale" must necessarily be one-dimensional. That is, one may also entertain the possibility that the resulting utilities are multi-dimensional in a sense that does not admit of direct comparisons and immediate substitutions between different good-and-penalties combinations. 2/ Finally, it is not clear that one can always safely assume the usual consistency axioms and thereby circumvent the need for formulating utility functions themselves on a probabilistic basis.

With the preceding discussion as background, we may now turn to the main suggestions of the present paper. First, we shall suggest that reduction to a single scale--hereafter called a "scalar-objective" approach--may not be adequate for many governmental and military purposes. Second, we shall suggest that it is desirable--or even necessary--to consider models, and related methods of analysis and computation, which effect, say, many of the stage-one and stage-two choices simultaneously, even under probabilistic conditions. Third, we shall also suggest that optimization considerations should continue to apply as pertinent features for judging any such model and its related modes of computation and analysis.

For an immediate illustration we may consider the acquisition of a new weapon system. It seems safe to say that one or more stages of

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1/ See, e.g., K. Arrow, [3] for a discussion of some of these basic (in principle) difficulties. See, also, however, S. Alexander [1] for a suggested way of dealing with some of these difficulties.

2/ Vide, e.g., Chapters XII and XIII by M. Hausner and R. M. Thrall, respectively, in R. M. Thrall, C. H. Coombs and R. L. Davis, [67] See also John Chipman, [28] as well as the more recent survey, from a behavioristic-and-organizational standpoint in Chapter 1 of M. W. Shelly, II, and G. L. Bryan, [60].

the decisions must involve probabilistic considerations. Simultaneity is also likely to be a feature that should be considered insofar as, say, costs and benefits are likely to be related not only to system and component designs but also to the quantity as well as the quality of the weapons and related support items that might be ordered. Finally, issues of optimality are also likely to be present. This is true even if multi-dimensional objectives--and related scalings--are being employed since without such optimality properties the resulting evaluations and substitutions may be erroneously made without reference to their "best" consequences on all parts of a proposed weapons-system complex.

We can perhaps make this last point somewhat more concrete by considering how the decisions for such a complex might be effected via suitably arranged simulations of a machine or hand variety. Presumably such simulations would make it possible to examine many possible arrangements of the component elements in order to assess--or evaluate--their consequences for system designs. But even this might not be wholly adequate unless related ideas of optimization are also employed. This need not exclude the possibility of continuing simulations, however, since such optimizations may also be used as guides or as controls.[1/] The point is, of course, that such guides or controls may be needed to reduce the likelihood of overlooking potentially valuable opportunities in any of the pertinent dimensions. These comments acquire

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1/ See in Chapter 5 of [10] for discussion of the use of such simultation controls in an example that involves a multi-dimensional objective (viz.,--the design of a network of city streets.)

even greater force at stages when design changes are to be effected and when risks of failure as well as levels of performance must be considered in a many-dimensional evaluation context.

We shall shortly provide other examples and illustrations. Commencing with one-dimensional scalar objectives we shall then want to continue in order, finally, to examine research possibilities and potential applications for models that are: (1) multi-dimensional in their measures sof effectiveness, essentiality, etc.; (2) adaptable to probabilistic contexts; and (3) have explicitly stated optimizing objectives either in their own right or because they are wanted as guides or controls for the simultations that are to be conducted.

3. <u>Some Examples of Chance-Constrained Programming with One-Dimensional Objectives</u>:

Chance constrained programming extends the ideas of ordinary linear programming to comprehend decision making in situations where probabilistic elements must be explicitly considered as integral parts of the model. Examples which may be suggestive of chance constrained programming and related approaches and possibilities[1/] are as follows:

1. PERT - Critical Path Analyses in which all or some of the times for the links associated with a project graph are known to vary statistically. This approach

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1/ E.g., stochastic programming and linear programming under uncertainty. Vide, e.g., [29] and [69].

then leads to a variety of new possibilities such as (1) explicitly determining a statistical distribution for the overall project completion times or (2) ensuring that the project--or subportions thereof--are completed within specified times at prescribed levels of probability. 1/

2. Choosing a course of studies and related R&D activities preparatory to introducing a new product. Here, too, a network or graph approach is utilized although these are inherently more complex than those utilized in the PERT or Critical Path Variety. 2/ For one thing, the network is not restricted to the simple dimension of "time" only. Different possibilities for marketing studies are considered along with their costs and related statistical qualities. Further supplementary conditions are also simultaneously considered along with the network (or graph) possibilities. These include the limits of allowable funds budgeted for the studies. They also include risk allowances stated, for instance, in terms of "payback periods" (formulated as chance constraints). 3/ Finally, the objective is also more complicated than the ones encountered in, say, PERT-type networks since (a) time alone 4/ is not a decisive consideration and (b) various priorities or "preemption" possibilities 5/ must be allowed for.

1/ See, e.g., [24]. See also [32], [40] and [49].

2/ See [19] and [20].

3/ Note that this use of payback as a risk constraint accords it the status of a "screening mechanism" and thus this treatment differs from the usual ones (described in the finance-and accounting literature) wherein payback is generally accorded the status of an "objective". (We shall make this terminology somewhat more precise in the immediately following sections.)

4/ E.g., time as interpreted via "payback period" constructs

5/ In the sense of a system of a "preemptive priorities" of the kind used in the literature, say, of queuing theory. See, e.g., pp. 237 ff. in T. L. Saaty [55].

3. Scheduling production, transportation and inventory to meet demands that can only be anticipated probabilistically--and to do so in a way that minimizes the expected sum of production, shipment and inventory carrying for a sequence of time periods.

   Here a utilization of chance constrained programming makes it possible to prescribe levels of quality and reliability of service and then to evaluate the consequences of altering any or all of these levels. 1/

4. Chance-constrained programming also offers an approach to queueing problems in which the queues may be partially controlled or influenced by managerial decisions. In this approach--unlike more usual queueing models--the decision rules and related optimizations are explicitly stated as an integral part of the model. A case in point involves scheduling tankers to a refinery in which inventory awaiting withdrawal must be simultaneously scheduled in the light of tanker availability and route reassignment possibilities. 2/ The explicit incorporation of an optimizing objective then makes it possible to provide evaluations that extend to altering docking and storage facilities as well as refinery and tanker capacities.

All of the above examples proceed by reference to a scalar objective. The issue of which objective should be elected thus becomes moot. We shall subsequently return to this topic in order to indicate some of the elections that are now available and then, by reference to an hypothetical military application, show some of the further multi-dimensional possibilities that might also be examined.

1/ See [13] and [21]. See also [51].

2/ Cf. [9].

4. <u>Some Terminological Definitions and Concepts</u>:

In this section our immediate purpose is to define and clarify some key terms and concepts and, to this end, we utilize a somewhat artificial and highly simplified example.

We may develop the idea of a "scalar objective" along lines like the following. Let v_i be the number of ships of class i that might be assigned to a particular "fleet" or "task" force. Let c_i, a known constant, be the cost per year[1/] of maintaining a ship of class i. These numbers c_i will hereafter be called <u>criterion</u> elements. We may also define a number g, a scalar, by the relation

$$(1) \qquad g = \sum_{i=1}^{m} c_i v_i = c_1 v_1 + c_2 v_2 + \cdots + c_m v_m$$

and then refer to this g as a "figure of merit"--i.e., a "numerical score" which results from any assignment of v_i values with the indicated c_i weightings.

Next we introduce the idea of a "constraint" as follows. Suppose we have a set of known constants a_{ij} and suppose we let each such constant represent the effectiveness of a ship in class i when it is assigned to mission j. These a_{ij} values might be rated relative to

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1/ This example is only illustrative, of course, and the criterion elements are here accorded the interpretation of "costs" only for concreteness. More complex weightings may be utilized and, if desired, the above concept of a "figure of merit" may be utilized even when, say, preemptive priorities and other such criterion elements are wanted even though the latter are not considered to be "measurable" in common numerical usages. See, e.g., [12] and [16].

some scale such as, say, a pure number scale ranging from 0-1,000. On this same scale we might then rate each of $j=1, \ldots, n$ missions by means of numbers t_j that represent the relative difficulty of each such "mission." By reference to these a_{ij} values, as given, we might then seek to effect ship assignments v_i that will accomplish all missions at the indicated difficulty levels t_j. By assuming that all constraints are given as linear inequalities we could then translate this into mathematical terms as follows: "Choose values v_i which satisfy "constraints" of the form

$$
(2) \qquad \begin{aligned} \sum_{i=1}^{m} v_i \, a_{ij} &\geq t_j, \\ v_i &\geq 0, \end{aligned}
$$

there being n such constraints, in all, that must be simultaneously satisfied.[1/]

A formulation such as

1/ Plus, of course, the m "non-negativity constraints," $v_i \geq 0$, $i=1, 2, \ldots, m$. We should also note that this may have to be accorded an integer programming--or mixed integer programming characterization when some of the v_i must refer to "whole ships." We do not discuss this topic here but refer rather to some of the surveys that are already available. See, e.g., A. Ben Israel and A. Charnes [6] as well as R. M. Young [74]. See also R. Gomory and W. J. Baumol [35] as well as H. M. Weingartner [73] for examinations of evaluational possibilities. For some recent research on algorithms see, G. L. Thompson [66] and F. Glover [34].

$$\text{minimize } g = \sum_{i=1}^{m} w_i c_i$$

subject to

$$\sum_{i=1}^{m} w_i a_{ij} \geq t_j \tag{3}$$

$$w_i \geq 0 ,$$

with $i=1, \ldots, m$, $j=1, \ldots, n$; would then produce a "linear programming model" [1] with the objective "minimize g." This, then, comprehends what we mean by a scalar objective--*viz.*, "optimize a specified (scalar) figure of merit." The term "optimize" is meant to include terms such as "maximize" and "minimize" and the related model is then referred to as an "optimizing model."

For further clarification we should note that none of this gainsays a use of other objectives such as, say, "secure a satisfactory figure of merit" and so on. Moreover, in our usage the term "objective" is not to be construed as being confined to linear programming models. It is meant to apply, rather, in any case where a figure of merit

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1/ We omit the further constraints which would probably be needed in a more realistic model. Specimen constraints which spring readily to mind are: (1) $\sum_{s \in S} w_s \leq W_S$ where W_S is a maximally available number of ships of a specified class and (2) $W_U \geq \sum_{k \in K} w_k \geq W_L$ where W_U and W_L are, respectively, upper and lower limits within which indices a_{ij} and t_j are valid approximations. Cf., e.g., Chs. X, XVI and XVIII in [12]. See also Chapters I and II in Ijiri [39].

is to be used in association with an explicit or implicit statement of how any result is to be judged.[1/]

We now note that the choice of a scalar objective implies the choice of a related figure of merit. It also carries certain further consequences as when, for instance, changes in a model and its related constraints are being considered. This leads up to a process which we shall refer to as "evaluation."

One way to perform "evaluations" is to effect trial and error substitutions of w_i values and observe the resulting figures of merit whenever some of the constraints are not fulfilled. This may also be extended to explorations of the g value consequences that might attend alterations in one or more constraints. Such a study might focus on variations of the initially specified a_{ij} in order to assess the desirability of changing the effectiveness ratings of one or more ships relative to the indicated tasks. Alternatively, variations in the t_j levels might also be assessed in a similar manner by reference to their potential effects on the figure-of-merit values, g.

For terminological convenience we may refer to the a_{ij} as "structural constants" and the t_j as "stipulations." Studies directed to exploring the consequences of altering stipulation values are sometimes referred to as "sensitivity analyses." We shall also

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1/ We again emphasize that these g values need not be assigned ordinary numerical interpretations. This is true even for linear programming or other optimizations that are not confined to so-called Archimedean fields. See, e.g., pp. 280-282 and pp. 755-756 in [12]. See also [60.1].

employ this term in the indicated manner and then distinguish it from a "general sensitivity analysis" in which both structure and stipulations are simultaneously varied in order to evaluate the changes in g values and related decision consequences.

When an optimization model of form (3) is available then the theory of linear programming permits another, more convenient, route for effecting a sensitivity analysis and related evaluations. This may be done by erecting a dual problem which recasts the data and by means of new variables and objectives proceeds directly to some of the wanted evaluations.

To make the sense of all this more concrete we now write such a dual to (3) as

$$\text{maximize} \quad z = \sum_{j=1}^{n} t_j x_j$$

$$\text{subject to}$$

$$(4) \qquad \sum_{j=1}^{n} a_{ij} x_j \leq c_i$$

$$x_j \geq 0$$

Here we may observe all of the previously noted features--*viz.*, the same data are utilized in association with a new set of variables. The objective is altered along with the sense of the inequalities, and, finally, the criterion and stipulation elements are interchange Cf. (3) and (4).

We now suppose that both (3) and (4) have optimal solutions which we may designate as w^* and x^*, respectively. The associated figures of merit may also be designated as g^* and z^* which, by virtue of the duality

theority of linear programming[1] produces $g^* = z^*$. Now suppose some subset of stipulations t_j are altered to new values $\hat{t}_j$ and then observe that none of these constants appear in any of the constraints (4). Thus, by reference only to the relevant criterion elements we can then obtain a new optimal figure of merit which we designate as $\hat{z}$. But again the theory of linear programming tells us that this will correspond to a new value $\hat{g}$ which is optimal for (3). The latter figure is based on the alterations from t_j to $\hat{t}_j$ in the relevant constraint. Also $\hat{g} = \hat{z}$ tells us that $\hat{g}$ is optimal. Hence, $\hat{g}$ presupposes that, _mutatis mutandis_, the preceding x^* values are all adjusted to new values $\hat{x}$ which are optimal for the thus altered constraints in (3).

We may now bring this discussion to its main point by observing that it is not necessary to determine these $\hat{x}$ values. Indeed the main point to observe is that one may proceed to explore the figure-of-merit consequences for (3) by varying only the t_j in (4) in the indicated manner. In each such variation[2] a _mutatis mutandis_ adjustment to a new optimum in (3) is thereby characterized. Hence a decision relative to t_j alterations may be reached on these presuppositions and the resulting trade-offs thereby evaluated in a readily ascertained and convenient manner.

1/ Cf. Chapters VI and VIII in [12].

2/ See [12] for discussions of how this may be done while allowing for alterations in the x values that are optimal for (4).

Of course, we have not yet said anything about how evaluations of the structural elements might also be accomodated under such an approach. For this we now refer to some of the results of relatively recent research because it may also help to clarify some of the remarks we entered under A.2 in section 2 above.

Thus suppose that a_{ij} and t_j values are given as in (2) above. Then to admit the possibility of changes in these structural and stipulation elements we now write

$$(5) \qquad \sum_{i=1}^{m} w_i \, (a_{ij} + y_{ij}) \geq t_j - u_j$$

as new constraints to be considered in place of the ones entered in (2). Here the y_{ij} represent potential increments to the initial ship-readiness (on mission j) values and the u_j represent potential decrements to the task difficulties. Thus, in particular, if we assign "costs" to these y_{ij} and u_j changes the problem becomes one not only of effecting ship assignments w_i but also of effecting, simultaneously, the indicated alterations in a_{ij} and t_j values in such a way as to meet a minimum cost objective.

We do not enter into further detail with respect to the constraints

(5) except to note that they are nonlinear. Furthermore, the costs of affecting y_{ij} and u_j changes are also likely to be nonlinear. Nevertheless, by virtue of relatively recent research, it is now known how to develop a corresponding <u>linear</u> programming model that will solve a problem with constraints like (5) in a way that meets the objective of this supposedly nonlinear problem. 1/ Of course, these statements are subject to qualification. We do not need to enter into this topic here except to note that enough has been accomplished to suggest that further research may make it possible to eliminate many of the stage-one and stage-two distinctions that we previously examined.

5. <u>A Stochastic Programming Approach</u>:

The above models were all deterministic and proceeded to optimizations that were accorded only a scalar objective and evaluation possibilities. Any such system characterization would evidently be of limited value in meeting requirements of the kind that are included in the following definition:

> "<u>System Effectiveness</u> is the probability that the system can successfully meet an operational demand within a given time when operated under specified conditions." 2/

This kind of requirement may, however, also be interpreted in terms of the constraint-objective and evaluation possibilities that we

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1/ See [17] for detailed discussion and explicitly stated qualifications that bear on the model equivalences implied by the above remarks.

2/ Source: p. 42 of NAVWEPS Report 8461, "Concepts Associated with System Effectiveness" (Washington, D. C.: Bureau of Naval Weapons, June, 1963).

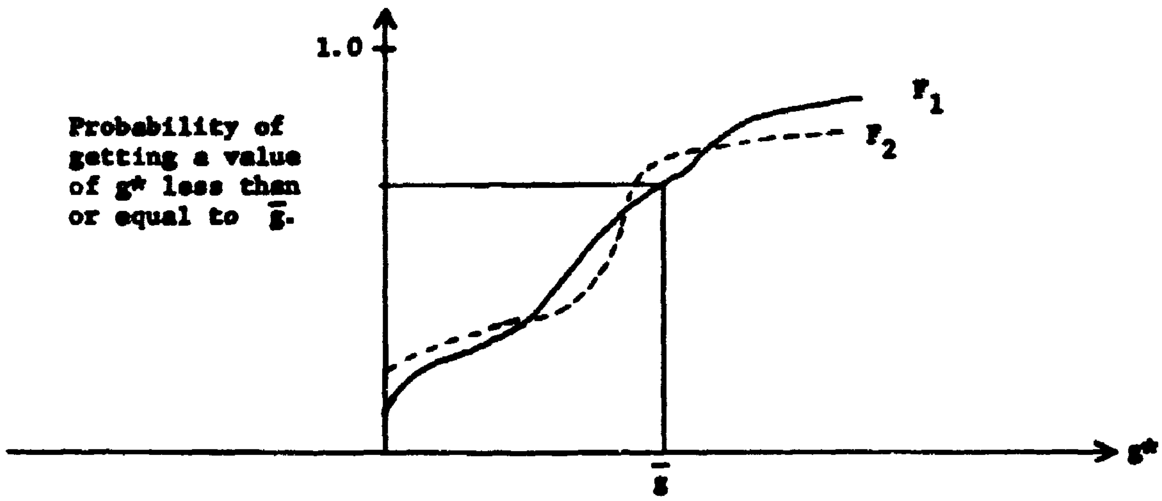
have already examined. Thus, refer to (2) and suppose that the constants are now random variables. For simplicitly, suppose, in particular, that the t_j ratings are known to be determined by some statistical distribution.

Recent research in "stochastic programming"[1/] suggests an approach that might be synthesized along the following lines. Choose a suitable Monte Carlo routine, say, to generate t_j values in accordance with the indicated statistical distributions.[2/] After each such set of values is generated apply known linear programming methods to effect w_i assignments that will optimally satisfy all constraints.

If g^* designates the resulting optimum figure of merit then we can continue with such Monte Carlo runs until a probability distribution is secured that looks like the solid line represented by F_1 in the figure below. From such a chart we can then, of course, determine the probability of securing an optimal figure of merit of value $\bar{g}$, say, or below.

1/ This term is due to G. Tintner. See, e.g., [68] and [69] and [70].

2/ Algorithms for certain classes of such models may now be developed from the analytical characterizations given in [24].



With respect to the scalar objective, g*, one now has an entire probability distribution for judging the results. This, as already indicated, provides a characterization in which one may estimate the probabilities of attaining different figure-of-merit levels, although, of course, to some extent it leaves aside issues like the following: 1/ (1) what is a "best" objective and (2) given such an objective how should the w_i assignments be made when the t_j values remain known only as random variables.

The research of Tintner and his associates has recently been directed to the latter issue. 2/ Something more may be immediately

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1/ Unless one wants to adopt the formalities of a utility theory approach as in [68] and [69]

2/ See [68], [69] and [70].

accomplished, however, if the t_j values can be submitted to choice and variation. Thus, for instance, a range of such values might also be shown for F_2 in this same figure. A comparison might then be made and a choice effected according to whether the probabilities associated with F_1 are preferred to those associated with F_2. See, e.g., [68], [69] and [59].

6. Constraint Violations and Chance-Constrained Programming:

In some cases we may be concerned with probabilities for fulfilling (or violating) one or more constraining condition. Here the research on chance-constrained programming may be of interest and so we proceed to point up this topic, too, by reference to the same simple example. For this purpose, then, we rewrite the constraint set (2) in altered form as follows:

$$\text{(6)} \qquad Pr\left[\sum_{i=1}^{m} w_i a_{ij} \geq t_j\right] \geq \alpha_j$$

Each of these $j=1, \ldots, n$ constraints is accorded interpretations like the following: "Pr" is an abbreviation for "probability" and every $0 \leq \alpha_j \leq 1$ prescribes a level of "confidence" or "reliability" which--by reasons of "policy," "technology," etc. 1/ --is considered desirable on mission j. That is, satisfaction of any such constraint

1/ Further development and refinement may be found in [21] as well as [51] and [52].

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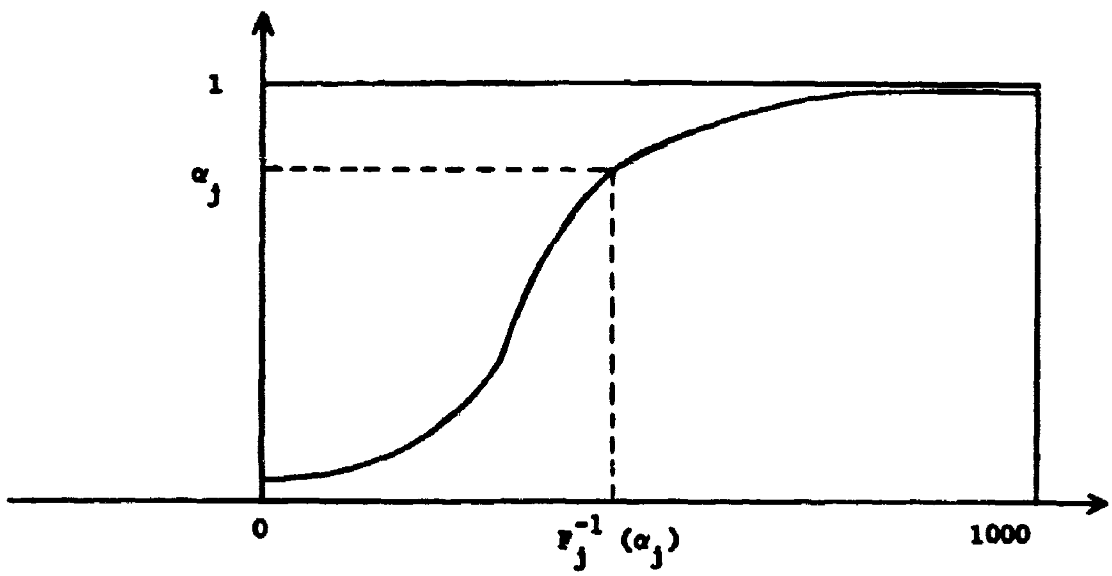
requires ship assignments to be sufficiently adequate so that the "risk" of falling below the corresponding difficulty index, t_j, will be at most $0 \leq \beta_j = 1 - \alpha_j \leq 1$.

A variety of approaches can be utilized for effecting ship assignments to meet such chance constraints. Depending on the context, one may designate a class of decision rules from which an optimum rule is to be selected. In general one may expect these optimal rules to be of a "conditional stochastic variety"--viz., some w_i assignments are contingently effected while others are witheld, or reserved, until the results of preceding assignments can be evaluated; then further w_i assignments and evaluations are effected; and so on.

Such conditional stochastic rules are comprehended under what are called the "higher-order decision rules" of chance-constrained programming. These are distinguished from the so-called "zero-order rules" in which the w_i assignments are all effected ab initio.

The latter class of rules generally leads directly to a fractile analysis for each constraint, as we now proceed to illustrate for (6). Thus, refer to the following figure. If the curve shown there corresponds to some function F_j interpreted as the marginal distribution of a random element t_j then the value $F_j^{-1}(\alpha_j)$ is the fractile corresponding to α_j. That is, this value, which is noted on the mission difficulty scale, prescribes the minimal effectiveness rating which the ship assignments, w_i, must achieve if the corresponding constraint in (6) is to be fulfilled with probability α_j.



To continue in this relatively simple class of cases and still extend the scope of our discussion we now replace the problem (3) by

$$\text{(7)} \qquad \begin{aligned} &\text{minimize } g = \sum_{i=1}^{m} w_i c_i \\ &\text{subject to} \\ &\qquad \sum_{i=1}^{m} w_i a_{ij} \geq F_j^{-1}(\alpha_j) \\ &\qquad w_i \geq 0 \end{aligned}$$

This is an ordinary linear programming problem. It is also called a "deterministic equivalent" for an original problem with chance constraints in the form (6). This term is meant to suggest that (a) no random terms appear in (7) and (b) any w_i values which optimally satisfy

this deterministic problem will also optimally [1/] satisfy the corresponding chance-constrained problem.

As has just been observed, (7) is an ordinary linear programming problem. Hence it has a dual which may be written [2/]

$$
(8) \qquad
\begin{aligned}
&\text{maximize} \quad z = \sum_{j=1}^{n} F_j^{-1}(\alpha_j)\, x_j \\
&\text{subject to} \quad \sum_{j=1}^{n} a_{ij}\, x_j \le c_i \\
&\qquad\qquad\qquad x_j \ge 0 \,.
\end{aligned}
$$

The optimal x_j's may again be used as evaluators but now they apply to the fractiles, $F_j^{-1}(\alpha_j)$, which are needed to insure the wanted reliability levels. When the figure of merit is as indicated in (7)--hence when the criterion elements are stated in cost per ship per year--then the x_j values represent the related dollar evaluations that are optimally imputed to the corresponding mission reliability levels, α_j, or, more precisely, to the fractile values with which they are associated.

The z value in (8), like the g value in (7) is stated in dollars although, of course, the criterion elements differ in that the c_i refer to ship costs whereas the $F_j^{-1}(\alpha_j)$ refer to the fractiles needed to ensure

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1/ Of course, this optimality property assumes that a zero-order rule is applicable.

2/ These reductions readily follow from a more general theorem due to A. Ben Israel. See [5] and [6].

accomplishments of the indicated missions at the specified probabilities α_j. Thus, in particular, the x_j's represent dollar values that are implicitly being assigned to missions, rather than ships, at the indicated probability levels α_j.

In some cases it may be desirable to replace these x values by a variety of non-dollar indexes. For instance, we may want an index which can be referred to the original effectiveness-mission difficulty scale. Such an index, which represents a weighted average of the already available mission reliability evaluations, can be developed as follows. Define a set of ratios

$$0 \leq r_j = \frac{x_j}{\sum_{j=1}^{n} x_j} \leq 1 \tag{9}$$

so that each ratio reflects the relative value imputed to mission j by the indicated assignments. Applying these results in a fairly obvious way, we would then have an overall index

$$R = \sum_{j=1}^{n} F_j^{-1} (\alpha_j)\, r_j \tag{10}$$

which can be referred to the original 0-1,000 scale to provide, *inter alia*, a measure of overall system effectiveness for the indicated combination of missions at the specified reliability levels.

<u>Remark:</u> Other possibilities might also be entertained via relatively straightforward extensions of the above approach. For instance one might proceed to maximize one or more combinations of system indicators while according the related dollar magnitudes only the status of a budgetary

constraint.[1/] In this connection it should be of interest to note that recent research has shown how any <u>ratio</u> of linear forms to be optimized (under linear inequality constraints) can be reduced and treated by the methods of ordinary linear programming. [2/]

We leave these topics aside, however, in order to examine other possible objectives which, <u>inter alia</u>, will also allow us to indicate some of the ways in which higher order decision rules may operate.

7. <u>Alternative Objectives</u>:

As already noted, the higher order rules of chance-constrained programming generally proceed in conditional stochastic fashion. I.e., a subset of w_i values is first designated and these resources only are committed until additional readings can be made; then a further subset of w_i values is designated and a new set of readings obtained; another subset of w_i values is then designated; and so on. In these cases one cannot proceed directly to a fractile formulation as in (7) and, furthermore, the objective must be stated in a way that reflects the statistical character of the optimizations and related constraint fulfillments that are being conditionally pursued.

The possibility of a rather wide choice of objectives is thereby opened and thus there is also opened a corresponding range of choices among cost-effectiveness measures and related evaluations. We illustrate some of these possibilities as follows.[3/]

1/ The dual evaluators associated with such constraints would then have the character of imputed rates of interest. See, e.g., [12] Chapters XV and XVII. See also H. M. Weingartner [73] as well as B. Naslund and A. Whinston [52].

2/ I.e, by reference to ordinary linear programming "equivalents" of these nonlinear models. See [14]. See also [15] for further references.

3/ See [11] from which these materials have been extracted.

Case i: "E Model." For this model the figure of merit g is defined by reference to

$$g = E\left(\sum_{i=1}^{m} w_i c_i\right) \tag{11}$$

where "E" means "expected value." An associated optimization then proceeds with respect to this figure of merit g as thus defined.

<u>Remark 1:</u> Although such "expected value optimizations" have often been criticized, 1/ they have nevertheless been widely employed. 2/ The point of many of these criticisms bears on the fact that in many situations reference must be made to "risks" which emanate from "skewness" and other such properties of the pertinent statistical distributions. 3/

<u>Remark 2:</u> Some protection against the full force of these criticisms can be secured by flexible use of the constrained optimizations being considered here. Thus, for instance, one might introduce associated penalty terms in the functional as is done in linear programming under uncertainty, for instance. 4/ Alternatively one can introduce constraints which bear on the allowable amounts of risk and refine these controls in a variety of ways. For instance, one may replace any t_j in (6) by a new random variable $\hat{t}_j = k_j t_j \pm a_j \sigma_j$ to indicate that only a certain proportion of this t_j value is to be considered, plus a further term which is additive. 5/ Also, one

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1/ See, e.g., [4] p. 39, for a summary discussion.

2/ See, e.g., G. B. Dantzig [29] and G. B. Dantzig and A. Madansky [30] for such objectives as utilized for "linear programming under uncertainty."

3/ See, however, the discussions in Chapter 6 of [38].

4/ Vide, e.g., [29] and [30]. See also the paper [22] for further illustrations as well as a demonstration that all two-stage models of linear programming under uncertainty can be reduced to a single type of deterministic equivalent.

5/ One can also permit these proportionality factors to be selected as a part of the optimization. See, e.g., C. Van de Panne and W. Popp [71].

can iterate any constraint to allow for still further controls by assigning different α_j values to each constraint in the indicated iteration. 1/ Finally, of course, one can combine penalty-term approaches with these constraint alterations and also introduce still further constraints for their control-and-evaluation possibilities. 2/

Case ii: "V Model": In some cases we may be concerned with an overall "risk minimization." One such approach would proceed via the so-called mean-square error defined by

$$g = E \sum_{i=1}^{m} (w_i c_i - w_i^o c_i^o)^2 \tag{12}$$

wherein the $w_i^o c_i^o$ represent some set of "preferred values." Minimization of the figure of merit g would then correspond to risk minimizations of the kind that are sometimes utilized in selecting investment portfolios 3/ and also (with suitable criterion elements) the kinds of optimizations that are sometimes pertinent for the design or operation of complex engineering systems.

Remark: Optimizations such as (12) may be undertaken in their own right or else they may be undertaken in conjunction with chance constraints in the form of (6). Some progress has been made in achieving deterministic equivalents for the latter class of cases. See [11]. Further research is also needed to obtain insight on where such objective-and-constraint combinations may best be employed. 4/ Thus, for instance, one might wish to continue with an "E Model" subject to a risk constraint of the form

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1/ See [11], p. 21, for further discussion.

2/ See, e.g., [13] for an example where an "horizon constraint" is introduced to evaluate the time interval over which a tanker-fleet acquisition program is being considered.

3/ See, e.g., [48].

4/ See, for instance, the remarks by Naslund and Whinston [52] for the risk-control and evaluation possibilities of an "E Model" in an investment analysis.

$$(13) \qquad E \sum_{i=1}^{m} (w_i\, c_i - w_i^o\, c_i^o)^2 \leq \hat{g}$$

where $\hat{g}$ is some fixed value --and, of course, still other possibilities are present.

Case iii: "P-Model". In this approach one might continue with the constraints (6) and then orient the figure of merit toward an α value as in

$$(14) \qquad \alpha \equiv \Pr \left[\sum_{i=1}^{m} w_i\, c_i \leq \sum_{i=1}^{m} w_i^o\, c_i^o \right]$$

and then proceed to "maximize α" as the objective.

Remark 1: It is of interest to observe that recent research 1/ has shown how to obtain deterministic equivalents for the objectives (11), (12) and (14) and that, for the statistical distributions studied, the deterministic equivalents of the former pair both appear as constraints in the deterministic model that appears as the equivalent for the maximization of (14) under chance constraints.

Remark 2: By adding further constraints to (6) in such forms as

$$(15) \qquad \Pr \left[\sum_{k} w_k\, c_k \leq \sum_{k} w_k^o\, c_k^o \right] \geq \alpha_o$$

it is possible to make contact with the "satisficing objectives" introduced into the behavioral sciences and organization theory literature by H. A. Simon. 2/ Such additional constraints are needed to invoke the search mechanisms that Simon prescribes when, for instance, $\max \alpha < \alpha_o$ or when some of the constraints in (15) are not compatible with others of the form (6).

1/ See [11].

2/ See e.g., Chapters 14 and 15 in [61] as well as the article [62] by H. A. Simon.

<u>Case iv:</u> "G Models." These are generalizations of the (multi-dimensional) goal-programming models of ordinary linear programming. 1/

A. One such approach involves penalty-and-reward features which may be suitably associated with some subset of the constraints (6). 2/ Thus, for instance, we may define new variables z_j^+ and z_j^- by means of the relations

$$(16)\quad z_k^+ = \begin{cases} 1 & \text{for } \sum_{i=1}^{m} w_i\, a_{ik} \geq t_k \\ 0 & \text{for } \sum_{i=1}^{m} w_i\, a_{ik} \leq t_k \end{cases} \qquad z_k^- \begin{cases} 0 & \text{for } \sum_{i=1}^{m} w_i\, a_{ik} \geq t_k \\ 1 & \text{for } \sum_{i=1}^{m} w_i\, a_{ik} \leq t_k \end{cases}$$

and then attain a corresponding scalar optimization as

$$(17)\qquad \min\ g = E\left(\sum_k r_k\, z_k^+ - p_k\, z_k^- \right)$$

where r_k and p_k are the relevant penalties and rewards for satisfying (or failing to satisfy) the constraints indexed by k.

B. Another approach proceeds directly to a joint maximization as in

$$(18)\qquad \text{maximize}\ \alpha = \Pr\left\{ \sum_{i=1}^{m} w_i\, a_{ik} \geq t_k \right\}$$

1/ I.e., models in which constraints are incorporated in the functional with the objective being oriented toward "coming as close as possible to meeting all of the indicated goals." See Appendix B and Chapter X in [12].

2/ This builds on work published by A. Charnes and A. Stedry. See, e.g., [25] and [26]. See also [27].

where the braces indicate that this maximization is to be jointly undertaken over all the constraints indexed by k.

C. Still another approach is via a "maximin objective" [1] as in

$$\text{maximize } \alpha$$

subject to

(19) $$\Pr\left[\sum_{i=1}^{m} v_i a_{ik} \geq t_k\right] \geq \alpha$$

with, of course, still other constraints in the form (6) to be considered. Finally, of course, one could evidently replace this by a weighted combination of the form

(20) $$\text{maximize } g = \sum_k r_k \alpha_k$$

which then becomes an alternative expected-value optimization with constraints in the form

(21) $$\Pr\left[\sum_{i=1}^{m} v_i a_{ik} \geq t_k\right] = \alpha_k$$

where these $0 \leq \alpha_k \leq 1$ are variables to be chosen, subject perhaps to ancillary conditions such as

(22) $$\alpha_k \geq \alpha_\ell$$

when relative (probabilistic) preferences are thus to be accorded the missions to which they are related. When, further, no mission probability is to be permitted to fall below a prescribed level, α_o, we can also write

(23) $$\alpha_\ell \geq \alpha_o \ .$$

1/ Observe that any v_i assignment fixes the maximum attainable α at a level corresponding to the lowest probability which is thereby assigned to any one of these k constraints. Hence the term "maximin objective" is appropriate.

> and thereby assure that the maximization in (20) is not purchased at an unacceptable level of risk for any of the indicated missions.

The above suggestions are not exhaustive. There are many possibilities. In the "G models", for instance, we have illustrated some of the joint constraint possibilities relative to the objectives. We can also introduce "joint constraints"[1/] *per se* in such forms as

$$\Pr_{k\epsilon K} \{ \sum_{i=1}^{m} w_i\, a_{ik} \geq t_k \} \geq \alpha \tag{24}$$

where now all $k\epsilon K$ of the indicated constraints are to be jointly satisfied at some prescribed level $0 \leq \alpha \leq 1$. For instance, if $K = \{1, 2\}$ then

$$\Pr_{k\epsilon K} \{ \sum_{i=1}^{m} w_i\, a_{ik} \geq t_k \} = \Pr \{ \sum_{i=1}^{m} w_i\, a_{i1} \geq t_1, \sum_{i=1}^{m} w_i\, a_{i2} \geq t_2 \} \geq \alpha$$

is the pertinent condition and the w_i assignment must jointly satisfy this constraint with probability of at least $\alpha 100\%$.

1/ Cf. [72].

8. K-Efficiency and Multiple-Objective Optimizations in Deterministic Models:

The above characterizations provide entrances to various ways in which multiple objective approaches might be used in association with chance-constrained programming. It will help to make matters somewhat more clear, however, if we do not proceed directly to the latter topic but enter it rather by means of only deterministic characterizations which we have elsewhere called by the name of K-efficiency.[1/]

To introduce these concepts we may suppose that

$$\sum_{i=1}^{m} w_i \, a_{ik} = T_k \tag{25}$$

defines a distinguished set of indices among the set $j=1, \ldots, n$ in (3). Each such T_k may be interpreted as a measure of effectiveness for the mission with which it is associated. For ease of exposition we may suppose that these k indexes are applicable to the first K constraints in (3).

We want an optimization that avoids assumptions of comparability between any of these effectiveness measures and this is done by means of K efficiency as follows. Let

$$T \equiv (T_1, \ldots, T_k, \ldots, T_K) \tag{26}$$

so that T is a vector in which comparisons can be made only between elements in the same position. Thus $\hat{T} = T$ if and only if $\hat{T}_k = T_k$ for

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1/ After Tjalling Koopmans who, at an early date, showed how these ideas of "Pareto optimality" could be formulated in terms of so-called "activity analysis models." See [42].

every $k=1, \ldots, K$ and $\hat{T} \geq T$ if and only if $\hat{T}_k \geq T_k$ for each individual index k.

Now consider the problem[1/]

$$\sum_{i=1}^{m} w_i a_{ik} = T_k , \; k=1, \ldots, K$$

$$\sum_{i=1}^{m} w_i a_{ij} \geq t_j , \; j=K+1, \ldots, n$$

(27)

$$\sum_{i=1}^{m} w_i c_i \leq C$$

$$w_i \geq 0, \quad i=1, \ldots, m$$

and suppose that a set of values $\hat{w}$ satisfies all constraints and produces a vector of values $\hat{T} = (\hat{T}_1, \ldots, \hat{T}_k, \ldots, \hat{T}_K)$. This vector $\hat{T}$ is "efficient" (or "optimal") if and only if there is no other w which also satisfies all constraints and produces a vector of objective values $T = (T_1, \ldots, T_k, \ldots, T_K)$ with the property

(28) $$T \geq \hat{T}, \quad T \neq \hat{T}.$$

This is to say that T is <u>not</u> efficient if another vector T can be found with the following 2 properties:[2/] (i) no component of T has a smaller value than its corresponding component in $\hat{T}$ <u>and</u> (ii) at least one component in T is greater than its corresponding component in $\hat{T}$.

1/ We have here replaced the objective (1) with a budgetary constraint limiting expenditures to a maximal value, C.

2/ It is assumed that all constraints are also satisfied by the solution w associated with T.

Bearing the significance of these non-comparability assumptions in mind we can nevertheless formulate a corresponding linear programming problem in the following form

$$\max \quad z = \sum_{k=1}^{K} z_k$$

subject to:

$$\sum_{i=1}^{m} w_i \, a_{ik} - T_k \qquad = 0\,, \quad k=1, \ldots, K$$

(29) $$\sum_{i=1}^{m} w_i \, a_{ij} \qquad \geq t_j\,, \quad j=1, K+1, \ldots, n$$

$$\sum_{i=1}^{m} w_i \, c_i \qquad \leq C$$

$$T_k - z_k = \hat{T}_k\,, \quad k=1, \ldots, K$$

wherein $w_i \geq 0$ all i and $z_k \geq 0$ all k. We can then observe the following: (i) $\hat{T}$ with components $\hat{T}_k$ will be efficient if and only if $\max z = \Sigma z_k = 0$. Furthermore, if $\hat{T}$ is not efficient then the solution associated with T_k^* will be efficient. 1/

We can see this readily enough if we observe characteristics like the following: $z^* > 0$ can only occur if some $z_k^* > 0$. But this in turn implies that, for this same $k, T_k > \hat{T}_k$ for the same indices k. On the other hand $T_k = \hat{T}_k$ is the only other possibility since every z_k is constrained to be non-negative. Hence $z^* = 0$ implies $T_k = \hat{T}_k$ every k and evidently $\hat{T}$ is efficient since otherwise $z^* = 0$ would not be maximal.

1/ Cf. Chapter IX in [12] for the relevant theorems and proofs as well as an algorithm (the "spiral method") for locating all such efficient points.

Although a scalar objective has been utilized in (29) this should only be regarded as an artifact for purposes of analysis (proof) and computation. For instance, if two different vectors $\hat{T}$ and $\hat{\hat{T}}$ were specified which produced values $\hat{z}^*$ and $\hat{\hat{z}}^*$ which were both non-zero then one could only say, in general, that neither was efficient. The fact that $\hat{z}^* > \hat{\hat{z}}^* > 0$ should not be automatically accorded any significance. The comparison is only with reference to efficient possibilities as when, for instance, $\hat{T}$ has some components which are larger and some which are smaller than their corresponds in $\hat{\hat{T}}$.

Similar remarks apply, of course, to the cases in which $\hat{T}$ and $\hat{\hat{T}}$ are both efficient and, indeed, one may generally expect many such efficiency possibilities in a realistic problem.[1] Direct comparisons between such efficient solutions is not possible without recourse to supplementary criteria. On the other hand, all such efficient solutions will be optimal in the sense that one cannot improve any T_k in an efficient T without worsening one or more of its other components. Thus, on the "design side"[2] when, say, $\hat{T}$ specifies a set of values $\hat{T}_k$, one for each objective, then one can be sure of the following when a z^* is secured via (28): (i) the solution associated with z^* is efficient, (ii) no design specification $\hat{T}_k$ is violated, and (iii) the trade-off terms for any other efficient possibilities can be readily ascertained.[3]

1/ See preceding footnote.

2/ In the sense of the stage-one and stage-two and intermediate stage characterizations that were discussed in section 2, above.

3/ E.g., via a dual evaluator analysis of the kind discussed in section 4. See, however, Chapter IX in [12] for further possibilities that may also be used.

Thus, in particular, these trade-off possibilities are optimal in the already indicated senses.

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9. Functional Efficiency and Multiple-Objective Optimizations for Chance-Constrained Models:

To deal with multiple-objectives in chance-constrained problems it is necessary to extend the above ideas and so we therefore invoke the more general ideas of functional efficiency.[1/] For this purpose we introduce $r = 1, \ldots, R$ functions

$$F_r \, (T_1, \ldots, T_K) \tag{29}$$

which may each represent, say, a "utility function" or some other suitable indicator of satisfaction. Again we do not wish to admit direct comparisons--at least until a "functionally efficiency" point has been achieved so that, then, we may be assured of optimality in each such dimension before proceeding to a trade-off analysis.

Hence proceeding to extend the analysis which has just been given we have recourse to an analogous artifact and write our problem in the form[2/]

$$\max \quad z = \sum_{r=1}^{R} z_r$$

$$\begin{aligned}
&\text{subject to} \\
&\sum_{i=1}^{m} w_i \, a_{ik} - T_k && = 0 \\
&\sum_{i=1}^{m} w_i \, a_{ij} && \geq t_j \\
&\sum_{i=1}^{m} w_i \, c_i && \leq C \\
&f_r \, (T_1, \ldots, T_k, \ldots, T_K) - z_r = f_r \, (T_1, \ldots, T_k, \ldots, T_K) \\
&w_i \geq 0, \; i = 1, \ldots, m \\
&z_r \geq 0, \; r = 1, \ldots, R.
\end{aligned} \tag{30}$$

1/ Also called "f_j-efficiency." See Chapter IX in [12] where these ideas were first presented.

2/ Giving a multi-dimensional utility optimization, if one is wanted.

Evidently, again, $\max z = z^* = 0$ if and only if $T = (T_1, \ldots, T_k, \ldots, T_K)$ is "functionally efficient." 1/

We need not here elaborate further on these ideas of functional efficiency per se since our purpose is rather to utilize these ideas in formulations like the following:

$$\max z = \sum_{s \in S} \delta_s$$

$$\text{subject to}$$

$$(31) \qquad \Pr\left\{ \sum_{i=1}^{m} w_i a_{ij} \geq t_j \right\} - \alpha_s \geq 0$$

$$\alpha_s - \delta_s = \alpha_s$$

where the chance constraints (which may perhaps be joint) 2/ that we have here singled out for attention all have their α_s values as variables to be optimally determined subject only to $0 \leq \alpha_s \leq 1$. To be sure, there will normally be other constraints with fixed α_j values. The point is, of course, that this is intended as an extension to a multiple-goal formulation in a probabilistic context which we now readily achieve by requiring $\delta_s \geq 0$ for all $s \in S$.

In some cases we may wish to consider preemptive priority possibilities in which, say, mission s_1 is of paramount importance. We handle this by means of the artifact: $M_{s_1} \gg M_{s_2}$ whenever $s_1 > s_2$ by which we mean there is no number k with the property $kM_{s_2} \geq M_{s_1}$. 3/

1/ Note, however, that this does not then require $T^* = T$ as in (29).

2/ Cf. (24) ff. See also [72]. We are here omitting cost and other constraints (deterministic or not) which would be needed to complete these models. See (28) and the footnotes on p. 17.

3/ I.e., these M values have the so-called non-archimedean order property. See, e.g., pp. 756 and passim in [12].

Then we write

$$(32) \qquad \begin{aligned} &\max\ z = \sum_{s_j \in S} M_{s_j}\, \alpha_{s_j} \\ &\text{subject to} \\ &\Pr\left\{ \sum_{i=1}^{n} w_i\, a_{ij} \geq t_j \right\} \geq \alpha_{s_j} \end{aligned}$$

and require only $0 \leq \alpha_{s_j} \leq 1$, all $s_j \in S$. Again we leave these α_{s_j} values to be determined by the optimizing choices of w relative to the preemptions.

To help underscore the significance of (31) we observe that some lower ranking α_{s_j} may be accorded values $\alpha_{s_j} = 0$. Indeed, even very large increases in these α_{s_j} may be foregone if any such increase must be purchased at the cost of even a very small decrement in any mission which has a higher preemption index.

Such preemptions may be too strong for many applications. But then, of course, it is possible to combine (31) and (32) in a variety of ways or else to proceed directly to minimum stipulations on pertinent missions as in the following:

$$(33) \qquad \begin{aligned} &\max\ z = \sum_{s_j \in S} M_{s_j}\, \delta_{s_j} \\ &\text{subject to} \\ &\Pr\left\{ \sum_{i=1}^{m} w_i\, a_{ij} \geq t_j \right\} - \alpha_{s_j} \geq 0 \\ &\alpha_{s_j} - \delta_{s_j} \geq \hat{\alpha}_{s_j} \end{aligned}$$

where, as in (31), $\delta_{s_j} \geq 0$, also applies.

10. <u>Conclusion</u>:

We have now surveyed a variety of possible approaches to cost-effectiveness that might be secured from the models and methods developed in recent research. This survey has not been exhaustive[1/] and, indeed, we have not even covered all of the possibilities even from chance-constrained programming.[2/]

In all except the last preceding section we have attempted to supply evaluations for possible use (including computational feasibility) along with our delineations of the possibilities that are now open. Further research on methods of computation for these nonlinear models will probably be needed. On the other hand, this need not be an impossible barrier to immediate use in all cases. Notice, for instance, that the preemptive priority model (32) admits of an approach like the following: Commence with the objective that has highest priority. Assign all available resources to this objective until a maximum probability is achieved. If any resources remain, proceed to the next preemptively ordered objective and assign the remaining resources to it. Check these assignments by examining whether alternative allocations will permit an increase in the latter without producing any decrease in the former. Continuing in this manner one may expect to deal in general with only a few such objectives at best.

1/ E.g., we have not adequately attended to all of the possibilities from stochastic linear programming and we have only barely mentioned the topic of linear programming under uncertainty.

2/ For instance we have not really explored the possibilities for Remark 2 under Case i in section 7 and, in particular, we have not explored the possibilities of joint determinations for t_j and α_j.

We may also note that analogous remarks apply for (33) and that even in the more difficult case of (31) the α_s values are likely to be fairly high. This means, in particular, that one will generally be confined to exploring only small regions in the tails of the associated statistical distributions.[1/]

We have also mentioned the possibility of different objectives along with the different evaluations that may result even when the corresponding optimal solutions are at hand. See section 7. Similar remarks also apply to the models covered in section 9 along with their resulting trade-off possibilities. Thus, unless a correct objective (multi-dimensional or otherwise) has been specified one may not have the "correct" trade-off possibilities even after an optimum has been attained. On the other hand, this need not be decisive. Thus, as in the case of the E-objective--see section 7 --the possibility of constraint adjunctions should not be overlooked. The possibility of model equivalences (even with different objectives) should also not be overlooked.[2/] Thus, even with completely different objectives, two different models may be equivalent in the sense that they lead to exactly the same decision.

This is all by way of saying that (a) the latter topics are also in need of further research and (b), in any event, work in 'model synthesis offers a fruitful way to clarify objectives in actual applications.

1/ Cf. the remarks in [21] for their bearing on "meaningful" policy conditions. See also the algorithms discussed there.

2/ See the concluding remarks in [11]. See also [16] and [12].

It thus seems appropriate to conclude by observing that there is now a need for the insight that can only be obtained from research conducted on such "probabilistic" models in the context of actual military problems. So far as we know only one such piece of research has been reported to date. This was the research directed toward the evolution of a multi-dimensional objective model of a chance-constrained variety that could be used as a guide for the IHAS (Integrated Helicopter Avianics System) designs under U. S. Navy auspices.[1/]

1/ Under Bureau of Naval Weapons Contract No. NOw 64-0138c. A published summary of this model and its multi-dimensional objectives may be found in [18].

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THREE LEVELS OF ANALYSIS OF COST-EFFECTIVENESS ASSOCIATED WITH PERSONNEL ATTITUDES AND ATTRIBUTES[1]

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The manager of any productive or service organization must see himself as a systems balancer (Bass, 1965). If he tries to maximize one particular output, say, immediate cash profits, or minimize some particular input, say, equipment costs, he is likely to run into trouble. For instance, his get-rich-quick efforts may inhibit the satisfactory maintenance or growth of his operations or generate labor unrest; his minimization of equipment expenditures may reduce the flexibility of his operations, again produce dissatisfied personnel and interfere with the quality of products and services.

As he tries to balance his system, the manager must deal with a multiplicity of inputs and outputs. In doing so, he seeks to satisfice on numerous objectives. Thus, in "off-the-record" interviews, 75 percent of 150 chief executives or their assistants mentioned more than one goal when asked, "What are the aims of top management in your company?" One-sixth mentioned more than three goals. They talked about growth, service, good products, making money, employee welfare, and so on (Dent, 1959).

If we wish to evaluate organizations involving the interplay of men, money and materials, organizations whose objectives are multiple, we must measure organizational performance on a series of dimensions involving human and material as well as monetary outputs. In the same way, if the cost-effectiveness of different organizations are to be compared (or the same organizations over time are to be studied), then human, material and monetary costs need to be taken into account. Again, multiple cost-effectiveness indices will be required with multivariate analyses of these indices to further understanding of the measures used. The single business firm can be studied at different points in time or as will be illustrated in this presentation, by multivariate comparisons with other business firms.

Analysis can proceed at three levels. Simple correlational analyses of the raw interacting financial, material and human resources may provide a surface empirical description of the multiple associations involved. The next and probably most popular level of analysis can be based on the interrelations among derived indices. Care must be taken here to avoid artifactual correlations.[2] But an even more abstract

level of study can be achieved by means of factor analysis which isolates a minimum number of fundamental dimensions underlying the surface correlations among the financial, material and human inputs and outputs among the systems under examination. With such factorial dimensions, we can describe the differential patterns which appear for different companies. Instead of evaluating cost-effectiveness of a firm on some single, simple good-bad, healthy-unhealthy, profitable-unprofitable or high cost-low cost measure, we become able to describe how a particular firm departs from the average for all companies under study on some minimum fundamental set of dimensions accounting for the common covariance among the companies on the multiple assessments employed.

Our purpose here was to apply the three levels of analysis to studying the interrelations among selected financial, material and sociopsychological measurements of the performance of 28 firms.

Basic Variables

Financial

The fiscal reports of 28 manufacturing companies provided sales and cost data from which to evaluate the differential effectiveness of the companies. The companies produced a variety of competing items for a common market with about the same numbers of employes, plant and capital. A company's market demand depended on how much more it advertised than its competitors and how much lower its prices were on each of the items it produced. The usual production, sales and cost figures were obtained for the reports of the 28 companies.

Attitudinal Variables

Just before the fiscal reports became public, a survey had been completed of the attitudes of company managements towards various issues about company operations. A parallel survey had been conducted among the workers within each of these companies.

The 13 questions to which managers and workers responded were as follows:

1. To what extent are you satisfied with the job you have been assigned?

2. To what extent is it easy to sell others in your company on new ideas?

3. To what extent is it easy to be open in expressing your real feelings on how operations are being conducted without being regarded as disloyal?

4. To what extent are you satisfied with your company's operations?

5. To what extent are you satisfied with your immediate boss?

6. To what extent is there overlapping of responsibility or duplication of effort?

7. To what extent are you satisfied with your subordinates?

8. To what extent are you clear about the goals of your job?

9. To what extent do you have an opportunity to make your own decisions rather than have them made for you by others?

10. To what extent do you have authority to make decisions which match your responsibility?

11. To what extent are there upward communication failures in the company?

12. To what extent do you feel caught in conflict of interests with other members in your company?

13. To what extent are decisions in your company made entirely by individuals at the top of the organizational hierarchy, by individuals located at all levels in the organization, by shared decision-making processes on an informal basis, by small groups at the top of the hierarchy or by groups representing all levels within the organization?

For each question, except the last, the individual respondent could answer using a nine-point scale: 9. A great deal; 8. Very much; 7. Fairly much; 6. More than some; 5. Some; 4. More than a little; 3. A little; 2. Hardly at all; 1. Not at all. A modified scale was used for the last question.

The mean attitude on each of 13 issues of the management and of the work force was calculated for each company.

Personnel Variables

Personnel records made possible calculations of the mean intelligence, personal orientations, age and work experience of both groups within each company. From personnel files were extracted verbal and numerical aptitude test scores of the managers and workers within each company; their age

and months of work experience were also obtained from these files along with their scores on a scale of attitudes towards management and labor. A high score indicated a pro-labor attitude; a low score was suggestive of a more conservative pro-management position. The scores of these managers and workers on an Orientation Inventory (Bass, 1962) yielded assessments of these individuals on the extent to which they were self-, interaction- or task-oriented. (A person high in self-orientation is rewarded by recognition and admiration from others; a person high in interaction-orientation is likely to be primarily rewarded by opportunities for having satisfying relations with others in the organization, while the task-oriented person is primarily pleased by opportunities for challenging jobs which he can complete successfully. The task-oriented person is most likely to persist in the face of failure.)

As before, the mean values for the managers of a particular company and the mean values for the workers of a particular company were calculated. These means were then correlated with cost-effectiveness and financial performance data for each of these companies.

Over 50 basic financial, social and psychological variables were collected for each of the 28 companies.

Methods of Analysis

Raw Data Level

Three levels of analysis were completed. First, we studied the matrix of the product-moment correlations among 52 variables. For instance, Tables 1 and 2 show portions of this matrix. Table 1 displays the correlations between the panel of financial variables with the panels of mean attitudes held by the managers and workers of the same companies. These simple correlations reveal which raw fiscal data appears to coincide with prevailing manager and worker attitudes in the same company. Table 2 shows the parallel first-order relations between financial data and personnel attributes of each of the companies.

We next moved from this empirical level to an examination of some logically derived variables.

Derived Index Level

Costs such as for raw materials obviously depended heavily on gross sales effectiveness. (The correlation was .84 between sales and raw materials costs for the 28 companies, for instance.)

Therefore, indices were derived logically to examine the relative costs per sale by the companies to see the costs of operation in the different companies with sales held constant. Profit indices were also figured here. These derived indices were then correlated with attitudinal information from managers and workers and with personnel attributes of the companies.

Eight cost-effectiveness indices were calculated:

1. Salary and Wages Costs Per Sales Income

2. Inventory Costs Per Sales Income

3. Advertising Costs Per Sales Income

4. Raw Materials Purchases From or Sales of Finished Goods to Other Companies Rather Than Direct to Market Per Total Sales Income

5. Cost of Materials Used Per Sales Income

6. Returns to Investors, Bonuses and Salaries and Wages Paid Out Per Net Worth (Aggregate Wealth Produced in Proportion to Original Investment)

7. Total Expenditures

8. Operating Profit (Before Taxes)

The correlations between attitudes and the derived indices are displayed in Tables 3 and 4.

Factor Level

This more abstract level of analysis was achieved by factor analyses of the matrices of raw intercorrelations among the financial, attitudinal and personnel variables. A factor analysis was completed for the financial variables alone, and another was completed for each of the managerial and worker panels of attitudinal variables for the 28 companies. Company policy was also interrelated with the variables. Half of the companies pursued a policy of "top-down" management with greatest attention on the external environment, centralized and individualized decision-making at the highest possible levels. The other companies pursued a reverse policy of greatest attention to internal affairs, decentralized and group decision-making at the lowest possible levels.

Each factor analysis provided a minimum number of orthogonal dimensions to account for the common variance among the variables in the

matrix so analyzed.[3] Instead of logically clustering the many variables we could measure, we empirically clustered into indices those variables high in correlation with each other and low in correlation with the variables of other clusters.[4] From this we could see how company policy related to performance on the abstract clusters of variables.

Results

Raw Data Level of Analysis

We will first look at how attitudes related to financial and material data, then how mean intelligence, age, orientation and so on were associated with monetary and material performance.

Attitudes Versus Financial Outcomes.

The first-order correlations among the raw variables of Table 1 gives us an empirical description of financial positions of the 28 companies which tended to match prevailing attitudes among managers and workers within the same companies.[5]

Satisfaction with one's particular job in the company did not seem particularly related to any significant degree with the financial performance of the company. On the other hand, satisfaction with the company operations (among both managers and workers) was greater in those companies who showed heavy use of raw materials (.47, .55) and large sales volumes (.51, .57). As might be expected, workers' satisfaction with company operations was heightened by higher salaries and wage payments (.43).

Management satisfaction with subordinates was higher when gross sales were high (.62) and the necessary raw materials costs (.56) associated with such sales were also high.

Both managers and workers saw much overlapping of responsibilities when gross sales were low (-.63, -.53) and less materials were used (-.66, -.65). At the same time, companies which failed to generate market demand through large expenditures for market research and advertising were likely to be seen by both their workers and managers as companies with much overlapping of responsibilities (-.47, -.35, -.37, -.49).

Ease in selling ideas had important association with gross sales (.48), raw materials used (.43) and low advertising costs (-.[illegible]8) for managers but not for workers. Yet, the ease with which managers or workers felt they could be open in criticism of the organization [illegible] unrelated to financial performance.

Worker satisfaction with boss seemed to be coupled with the readiness of the company to use large amounts of raw materials (.47), while managers seemed to be more pleased with their boss when advertising costs were high (.40) and only regular purchasing channels were used (-.40).

Clear job goals for both managers and workers were associated with sales (.38, .48) and raw materials used (.47, .50), but the opportunity to make one's own decision and to perceive that one's authority matched one's responsibility was unrelated generally to financial performance.

While upward communication failure was associated by both managers and workers with low sales income (-.35, -.42) and low materials usage (-.45, -.34), workers tended to see much greater upward communication failure when expenditures for equipment was kept low (-.59). Workers also perceived much more conflict of interest when equipment was not rented or purchased in large amounts (-.62). On the other hand, management perceived conflicts of interest to be high when gross sales activity was low (-.53), coinciding consequentially also with the lack of expenditures to cover such sales including expenditures for raw materials (-.55), for market research (-.41) and for advertising (-.45). Managers saw individual decision-making coinciding with low expenditures for equipment (-.64), while workers saw individual decision-making appearing in companies when gross sales were high (.37) along with expenditures for advertising (.71).

Of the nine raw financial items examined, only interest charges failed to be associated significantly with some mean management and/or worker attitude towards the company or some aspect of work.

Personnel Attributes.

Inspection of Table 2 reveals that the mean verbal aptitude of both managers and workers was strongly associated with the various financial outcomes. When the verbal aptitude of the average manager was high so were the company's gross sales (.48), raw materials used (.45), market research costs (.68), advertising costs (.47), and interest charges (.59). But as was subsequently discovered, those companies whose managers were high in verbal aptitude were also likely (.47) to contain workers similarly high on the average in verbal aptitude. Consequently, the mean verbal aptitude of workers also correlated positively with most of the same financial outcomes. When the mean verbal aptitude of workers was high, so was gross sales (.52), raw materials used (.59), salaries and wages (.51), storage charges (.42), market research (.47) and interest charges (.63). (The management-worker correlational differentials here tempt one to speculate that high managerial verbal aptitude particularly increased willingness of management to spend money on market research and advertising, while high verbal aptitude among workers increased the required wage and salary payments within the company as well as expenditures for inventory.)

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On the other hand, the mean quantitative aptitude of managers was unrelated to financial outcomes, although the quantitative aptitude of workers correlated with gross sales (.46), raw materials used (.46), salaries and wages (.47), and advertising costs (.41). Older workers, in particular, tended to push up raw materials usage (.53), storage charges (.49), and interest charges (.37).

The mean orientation of managers and workers seemed unrelated generally to financial outcomes to any significant degree although high self-orientation among workers coincided with high storage costs (.53). More strongly pro-union attitudes among workers led to higher sales (.48), higher salaries and wages (.42), higher equipment costs (.37) and higher storage charges (.56). More liberal union attitudes among managers tended to coincide with low storage charges (-.49), but higher market research (.76) and advertising (.52) expenditure. When the mean amount of managerial experience was high, so was raw material usage (.51) and interest charges (.37) while the amount of worker experience correlated .42 with raw material usage and .57 with storage charges.

<u>Derived Index Level of Analysis</u>

We can obtain an increased clarification of the meaningfulness of the first-order relationships observed in Tables 1 and 2 by turning to the more complex associations uncovered when the eight derived indices were correlated with personnel attitudes and attributes.

<u>Attitudes versus Indices.</u>

As seen in Table 3, none of the attitudinal variables correlated significantly different from zero with the first index, <u>wage cost per sales income</u>. <u>Materials used per sales income</u> also seemed to be independent of attitudinal data except for the significant correlation of .43 with the extent workers saw themselves in a conflict of interests and a negative correlation of -.59 with the extent to which they saw that decisions were made by individuals rather than by groups in the company. Thus, workers saw more conflict and more decision by groups rather than individuals in those companies which tended to use an unnecessary amount of materials in production for a given amount of sales. But management attitudes were unrelated to this particular index.

Generally, companies with high <u>inventory costs per sales income</u> were seen unfavorably by both company management and company workers. If inventory costs were excessive, satisfaction with company operations was low both among managers and workers (-.62 and -.56). There also was greater dissatisfaction with subordinates (-.56 and -.46). Furthermore, workers saw themselves with less clear job goals (-.58) while management saw much communication failure upward (.58), overlapping responsibilities (.46) and lack of ease in selling ideas (.46) when inventory costs per sales income were high in companies.

Unfavorable management (but not worker) attitudes towards the company and its operations tended to be associated with the extent to which the company was forced to buy and sell raw materials and finished goods to its competitors rather than carrying on all of its business in the regular channels of commerce. Presumably, the company that planned its production satisfactorily would have little need to turn to its competitors for assistance. While this activity outside the regular marketing and purchasing channels was unassociated with worker attitudes, Table 3 shows that where such outside-of-regu r-marketing activity was relatively high in contrast to the total sales volume for a company, the company management was likely to be unclear about its job goals (-.59), dissatisfied with its subordinates (-.49), feel that it was not easy to sell one's ideas to others in the organization (-.47), see a great deal of upward communication failures (.34), see little opportunity to make one's own decisions (-.43) and be dissatisfied with one's boss (-.40).

The financial index which seemed to covary most closely with both management and worker attitudes was advertising costs per sales income. Here again, an excess of advertising per sales dollar might come about not necessarily as a function of lack of advertising quality, but as a consequence of the failure of the company to produce the quality and quantity of products generating the sales income possible as a consequence of the amount of money spent on advertising. When advertising costs per sales income were excessive, management was highly dissatisfied with company operations (-.74) and with its subordinates (-.55). Management also saw much upward communication failure (.63) and a great deal of overlapping of responsibilities (.41). At the same time, excessive advertising costs were likewise associated with worker dissatisfaction with company operations (-.71) and confusion about job goals (-.71). The average worker also saw a high degree of conflict of interests and much upward communication failure (.53 and .52) when advertising costs were excessive. But opposite to the association with excessive materials usage, here workers were more likely to associate individual rather than group decision-making with excessive advertising costs.

Returns to investors, managers and employes, in the aggregate, were also reflected (positively, of course) in both management and worker attitudes toward the company. Both managers and workers felt more satisfied with company operations when returns were high (.55 and .62). They saw little overlapping of responsibilities (-.58 and -.49). Moreover, managers were more satisfied with their subordinates (.60) and with the ease in selling ideas (.39) in those companies whose returns and compensations were high. They also saw less upward communication failures and less conflict of interests (-.43 and -.43). At the same time, workers were more likely to have clear job goals (.48) and be more satisfied with their boss (.43). For managers but not workers, high gross returns on investment were more common when decisions were seen as made by individuals (.40).

Cost-Effectiveness versus Personnel Attributes.

As can be seen in Table 4, high wage costs per sales dollar were more likely when workers were higher in overall verbal and quantitative aptitude (.41). In turn, intelligent workers pushed up total company expenditures (.56); thus, although total wealth produced was higher, operating profits were not when the average worker was high in aptitude. Total expenditures were also greater when the average manager was higher in intelligence (.51). On the other hand, older managers were less likely to go outside regular market channels relative to their total sales income of their companies (-.39). The same was true if workers were older (-.37). While attitude toward unions by managers was unrelated to cost-effectiveness indices, pro-union proclivities on the part of the workers were associated with low advertising costs per sales income (-.43), low material usage per sales income (-.64), greater returns on investment (.47) despite the larger amount of total expenditures (.49). Experienced managers had lower advertising budgets relative to their sales income (-.37) while companies with experienced workers had greater gross returns on investment and compensation to managers and workers (.38). Orientation of managers and workers was unrelated to any significant degree with any of the financial cost-effectiveness indices.

In sum, wage and material costs per sale do not reflect management and worker satisfaction as well as do inventory and advertising costs per sales dollar. Managers seem particularly dissatisfied in firms with much extra market activity. More able, liberal and experienced workers are found in firms yielding greater overall output, but they obtain higher wages so that higher profits do not follow. Verbally apt and liberal managers seem willing to spend more money likewise inhibiting any enhancement of profits.

Abstract Level of Factor Analysis

This more abstract level of analysis was reached by means of factor analysis of each of the panels of financial and attitudinal variables searching within each panel for a minimum number of dimensions to account for the common variance among companies in their fiscal and morale standings. Table 5 displays the correlations among the raw financial variables. Table 6 shows the four orthogonal, rotated factors which alone could account for approximately 85 percent of the common variance of Table 5 among the financial variables and the company policy. It also shows the factor loadings or the correlations between variables and factors.

The first factor, F1, was identified as Gross Processing Activity. Companies high on this factor had heavy sales to market (.87), associated heavy advertising costs (.87), market research (.72) and raw

materials usage (.80). Such companies also paid out higher salaries and wages (.56) and borrowed more money requiring greater interest charges (.49), "top-down" policies coincided with higher activity (.54).

Factor F2 was composed of such Variable Costs as wages (.74), storage (.74) and off-market purchases of raw materials or finished goods from competitors to meet market quotas (.72).

Factor F3 was seen as involving Conservative Planning, for companies high on this factor borrowed less money, thus had less interest costs (-.65), but produced more goods than they advertised or priced for in the market so had to sell to competitors. They probably underestimated their productive capacities and did not keep as well abreast of market conditions as their competitors. In line with the usual findings about the greater riskiness of decisions made by groups, the individualized "top-down" policies were associated with conservative planning (.48).

Factor F4 was heavily loaded on equipment costs. Companies with such heavy capital investments or leasing commitments tended to do more market research (.45) and possibly needed to turn to competitors on occasion for raw materials to purchase (.35).

Looking at the individual variables, one could see that sales to market, sales to other companies, equipment and advertising costs were "pure" measures each loaded on only a single factor. On the other hand, the other variables were more complex factorially. Raw material usage was high when gross processing activity was up as well as in companies with higher variable costs. The same was true (although the proportions were different) for salaries and wages. High storage costs coincided with high variable costs in general as well as liberal planning. Market research loaded on two or three of the factors as did interest charges. "Top-down" policies coincided with greater processing activity and with conservative planning.

Attitudinal Factors. The matrix of intercorrelations among the 15 managerial variables (not shown) was factor analyzed as was the matrix of worker variables[6]. Table 7 shows the varimax solution achieved for the managerial variables accounting for 74 percent of the common variance of the intercorrelation matrix. Table 8 shows the solution achieved for the worker variables accounting for 81 percent of the common variance.

The four managerial dimensions extracted and the variables with highest loadings (signs reversed where necessary) were:

M1 Satisfaction with Top Management and Policies

.85	Top-Down Policies
.72	Satisfaction with Boss
.55	(Lack of) Conflict of Interest
.46	Clarity of Job Goals

Although individual responsibility and internal responsivity were stated company policies, such responsibility and responsivity were not perceived by managers as associated with company policy. They contributed to variance on other factors, however.

M2 Satisfaction with Individual Responsibility and Authority

.90	Felt Opportunity for Own Decisions
.86	Felt Matching of Authority and Responsibility
.65	Group, Not Individual Decisions
.54	(Lack of) Upward Communication Failures

M3 Internally Responsive Company

.85	Perceived Internally Responsive Company
.51	(Lack of) Overlapping Responsibilities
.49	Individual, Not Group Decisions
.45	(Lack of) Conflict of Interests

M4 Satisfaction with Climate and Operations

.89	Satisfaction with Company Operations
.85	Satisfaction with Subordinates
.75	Ease in Selling Ideas
.71	Ease in Openness
.69	Job Satisfaction
.66	Clarity of Job Goals
.60	(Lack of) Upward Communication Failures
.58	(Lack of) Overlapping Responsibilities

Five factors were extracted from the mean responses of the workers to the questionnaires in the 28 companies. For workers, salient factors clustered around freedom from conflict (W1), permissive supervision (W2), satisfaction with working conditions, and as with managers, satisfaction with responsibility and authority (W5); but where the "top-down" policy was associated in a single factor with various aspects of managerial satisfaction (M1), the "top-down" policies in companies generated no such associated high satisfaction among workers whose only consistent perception was of much individual decision-making when operating under such stated policies.

Other Analysis Possible

We could proceed to obtain factor scores[7] for companies in each factor to profile companies on these abstract orthogonal dimensions. Or, we could correlate these scores across different panels to look somewhat more abstractly at the relations examined in the first four tables. However, such extensions are simply further exercises in the use of factor analysis and need not be pursued here.

Source of Data

Although the sources of the data are irrelevant to the methodological purposes of this paper, it may come as somewhat of a disappointment to learn of the sources of the data analyzed. Unfortunately, companies are not inclined to collect or publish such non-financial data as we examined here-- although such data may be available or requested by outside auditors.

The sources of the data were 28 "companies" each containing 16 graduate business students, half of whom served as managers and half as workers. The companies existed for four hours of production and competitive marketing, during which time the eight managers of each company purchased, planned, advertised, priced and sold five kinds of "I-beams" literally produced from cut-assembled-and-stapled IBM cards by the eight workers of each company. Real work was done by all; only the money was funny. The communication problems of the large organization were simulated by enforcing a rule that no more than five members of the company could ever meet face-to-face.[8] Nevertheless, the kinds of data examined here are identical to what can be analyzed in real companies, agencies or departments if it is possible to collect data. More extraneous variance is likely to appear in matrices of data from real life, but this is one more reason for suggesting the utility of multivariate analyses, such as presented, to tease out the interplay among financial, material and human variables affecting the systems under study.

Footnotes

[1]Supported by the Office of Naval Research, Group Psychology Branch, this report was prepared for the Proceedings of the Logistics Research Conference, Department of Defense, Warrenton, Viriginia, May 26, 27, 28, 1965. Reproduction in whole or in part is permitted for any purpose of the United States Government.

[2]Thus, sales and profits are artifactually related if profits are equal to sales less expenses. We correlate here two expressions, both of which contain or are entirely made up of the same variable.

[3]These factors were extracted by the method of principal components. In essence, a plotting of company standings on the 10 financial variables formed a hyperellipsoid of 10 dimensions. Factor analysis determined the relative lengths of its 10 axes. Little unexplained common variance was left after the first four longest axes had been noted; the other six were discarded. To simplify the structure, the four-dimensional ellipsoid was now rotated to achieve a varimax solution, a solution in which the variance is maximum of the factor loadings. This maximum solution distributes the variance each variable had in common with the other variables (its communality) so that the variable correlates as high as possible with as few factors as possible and as low as possible with all other factors. The factors are identified by the variables which alone or in combination correlate with the factors.

[4]Combining variables by factor analysis rather than combining them logically into indices helps to avoid the possibilities of misinterpreting the behavior producing the high or low index numbers. For instance, a materials cost/sales ratio could be low primarily because of lower-than-ordinary materials usage in production. Or it could be low because of higher-than-ordinary sales volume. Furthermore, sales volume could be high (lowering the ratio) as a consequence of the high production of underpriced goods or the low production of overpriced merchandise. At the same time, more materials would need to be used for the higher production of underpriced items raising the ratio. Given the right variables, factor analysis could help sort out companies according to their relative performance on whichever dimensions really underlay the firm-to-firm variations in materials usage, sales volume, pricing and production.

[5]For the 28 companies, a product-moment correlation of .47 would occur only once in a hundred times by chance. A correlation of .37 would occur about five times in a hundred by chance.

[6]Included in these analyses were mean responses to the 13 attitudinal questions, whether or not the company had a "top-down" policy and the mean response to a question as to whether the company was primarily responsive to the needs of its own members or to the external environment.

[7] To find a company high in standing on a factor, we sum the products of the company's standard scores on each of the original variables multiplied by their respective factor loadings. The factor loadings are the partial regression coefficients on the factor of each of the variables since the factors are orthogonal so that the multiple correlation between a Factor F and variables V loaded on it becomes:

$$R^2 = r^2_{FV_1} + r^2_{FV_2} \cdots\cdots + r^2_{FV_j}$$ and a company's factor score,

$$f = r_{FV_1} X_1 + r_{FV_2} X_2 \cdots + r_{FV} X_j$$ where X is the standard score of the company on a given variable.

[8] For details of UPPOE, the University of Pittsburgh Production Organization Exercise, see Bass (1964).

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TABLE 1

MEAN ATTITUDES OF MANAGER AND WORKERS RELATED TO
THE YEAR-END FINANCIAL REPORT OF 28 UPPOE COMPANIES

MEAN ATTITUDES	Gross Sales	Materials Used	Salaries & Wages	Equipment Rentals Depr.	Storage Charges	Off-Market Purchases	Market Research	Advertising	Interest
Managers									
Job Satisfaction	.37°	.25	.16	.22	.02	-.15	.04	.34	-.12
Ease in Selling Ideas	.48*	.43°	.24	.27	-.17	-.25	.23	-.38°	-.08
Ease of Openness	.30	.36	.17	.41°	-.21	-.08	.10	.16	-.16
Satis. with Company Operations	.51*	.47*	.31	.35	-.27	.06	.14	.15	-.18
Satis. with Boss	.14	.24	.22	-.12	-.12	-.40°	.21	.40°	.13
Overlapping of Responsibility	-.63*	-.66*	-.41°	.00	.12	-.13	-.37°	-.49*	-.11
Satis. with Subordinates	.62*	.56*	.31	.21	-.30	-.11	.40°	.36	.06
Clarity of Job Goals	.38°	.47*	.29	.01	.02	-.32	.02	.38°	-.21
Opportunity for Own Decisions	-.19	.01	-.11	.24	.15	-.18	-.40°	-.42°	-.35
Extent of Dec Matching Resp.	-.15	.02	-.13	.16	.15	-.22	-.31	-.43°	-.29
Upward Communication Failures	-.35	-.45°	-.30	-.27	.20	-.01	-.10	.01	.16
Extent of Conflict of Interests	-.53*	-.55*	-.31	.04	.18	.08	-.41°	-.45°	-.29
Individual, Not Group Decisions	.37°	.22	.34	-.64*	-.06	.15	.30	.40°	.14
Workers									
Job Satisfaction	.35	.31	.11	.05	-.06	.16	.15	.26	.11
Ease in Selling Ideas	.15	.30	.20	.21	.05	.27	.24	.06	.10
Ease of Openness	.33	.35	.25	.26	-.34	.40°	.35	.18	.17

TABLE 1 (cont'd)

MEAN ATTITUDES OF MANAGER AND WORKERS RELATED TO
THE YEAR-END FINANCIAL REPORT OF 28 UPPOE COMPANIES

MEAN ATTITUDES	Gross Sales	Materials Used	Salaries & Wages	Equipment Rental & Deprec.	Storage Charges	Off-Market Purchases	Market Research	Advertising	Interest
Workers (cont'd)									
Satis. with Company Operations	.57*	.55*	.43°	.27	-.22	.23	.26	.19	-.01
Satis. with Boss	.35	.47*	.36	.19	-.15	.26	.06	.12	-.12
Overlapping of Responsibility	-.53*	-.65*	-.43°	-.25	.27	-.07	-.47*	-.35	-.03
Satis. with Subordinates	.34	.36	.25	-.05	-.22	.20	.34	.15	-.02
Clarity of Job Goals	.48*	.50*	.21	.36	-.48*	.18	.45°	.14	.00
Opportunity for Own Decisions	.24	.22	-.07	.16	-.33	.28	.42°	.20	.09
Extent of Dec. Matching Resp.	.22	.20	-.16	.18	-.27	.08	.32	.28	.22
Upward Communication Failures	-.42°	-.34	-.33	-.59*	-.25	-.24	-.32	-.17	-.01
Extent of Conflict of Interests	-.28	-.08	-.16	-.62*	.12	-.29	-.12	.02	.27
Individual, Not Group Decisions	.37°	.06	.15	-.32	.04	.09	.32	.71*	.19

*N = 28, d.f. = 26, $p < .01$ when $r = .47$
°N = 28, d.f. = 26, $p < .05$ when $r = .37$

TABLE 2

MEAN PERSONNEL ATTRIBUTES OF MANAGER AND WORKERS RELATED TO THE YEAR-END FINANCIAL REPORTS OF 28 UPPOE COMPANIES

PERSONNEL ATTRIBUTES	Gross Sales	Materials Used	Salaries & Wages	Equipment, Rental & Depreciat.	Storage Charges	Off-Market Purchases	Market Research	Advertising	Interest
Managers									
Age	.26	.44°	.09	.15	.02	-.20	.17	.03	.33
Verbal Aptitude	.48*	.45°	.36	.23	-.09	.23	.68*	.47*	.59*
Quantitative Aptitude	-.06	.03	.16	.01	.16	.17	-.04	-.06	-.06
Verbal and Quantitative Aptitude	.35	.40°	.40°	.13	.10	.21	.55*	.39°	.44°
Self-Orientation	.23	.11	.22	.12	-.09	.22	.36	.34	.05
Interaction-Orientation	-.15	-.07	-.04	-.40°	.29	-.28	-.30	-.08	.05
Task-Orientation	-.23	-.20	-.26	.20	-.35	.09	-.12	-.34	-.26
Liberal Attitudes to Unions	.36	.28	.05	.31	-.49*	.10	.76*	.52*	.28
Amount of Work Experience	.32	.51*	.15	.18	-.01	-.14	.24	.05	.37°
Workers									
Age	.28	.53*	.35	-.11	.49*	-.18	-.04	.22	.37°
Verbal Aptitude	.52*	.59*	.51*	.13	.42°	.06	.47*	.32	.63*
Quantitative Aptitude	.46°	.46°	.47*	.21	.27	.14	.30	.41°	.25
Verbal and Quantitative Aptitude	.48*	.54*	.53*	.24	.38°	.21	.39°	.29	.47*
Self-Orientation	.11	.14	.26	.17	.53*	.12	-.08	.05	-.13
Interaction-Orientation	-.16	-.24	-.23	-.13	-.27	-.07	.05	-.25	.26
Task-Orientation	.11	.12	.03	.02	-.14	.00	.12	.19	-.02
Liberal Attitudes to Unions	.48*	.30	.42°	.37°	.56*	.35	.36	.25	.27
Amount of Work Experience	.26	.42°	.36	-.01	.57*	-.08	-.14	.11	.35

*N = 28, d.f. = 26, $p < .01$ when $r = .47$
°N = 28, d.f. = 26, $p < .05$ when $r = .37$

TABLE 3

MEAN ATTITUDES OF MANAGER AND WORKERS RELATED TO THE COST-EFFECTIVENESS OF 28 UPPOE COMPANIES

MEAN ATTITUDES	Cost Effectiveness Index: Wage Cost Per Sales Inc	Inventory Per Sales Income	Advertising P. Sales Income	Extra Market Activity Per Sales Income	Materials Used P. Sales Income	Gross Return on Investment Plus Compen.	Total Expen-ses	Operating Profit
Managers								
Job Satisfaction	-.02	-.12	-.22	-.25	-.26	.27	.31	.33
Ease in Selling Ideas	-.04	-.46°	-.35	-.47*	-.28	.39°	.42°	.41°
Ease of Openness	.03	-.30	-.33	-.25	-.07	.24	.27	.26
Satis. with Company Operations	.03	-.62*	-.74*	-.31	-.25	.55*	.36	.57*
Satis. with Boss	.20	-.21	-.22	-.40°	.14	.03	.29	-.16
Overlapping of Responsibility	-.11	.46°	.41°	.22	.15	-.58*	-.54*	-.57*
Satis. with Subordinates	-.09	-.56*	-.55*	-.49*	-.25	.60	.46	.67*
Clarity of Job Goals	.11	-.28	-.15	-.59*	.08	.36	.34	.31
Opportunity for Own Decisions	-.10	-.18	-.21	-.43°	.03	-.00	-.32	.08
Extent of Dec. Matching Resp.	-.16	-.23	-.29	-.44°	.02	.02	-.29	.13
Upward Communication Failures	-.10	.58*	.63*	.34	.14	-.43°	-.26	-.37°
Extent of Conflict of Interests	-.04	.33	.25	.25	.17	-.43°	-.51*	-.38°
Individual, Not Group Decisions	.24	-.38°	-.04	.27	-.08	.40°	.41°	.17
Workers								
Job Satisfaction	-.20	-.31	-.30	-.03	-.28	.33	.23	.43°
Ease in Selling Ideas	.10	-.07	-.15	.08	.01	.17	.16	.09
Ease of Openness	.09	-.16	-.23	.22	-.11	.29	.31	.25

TABLE 3 (cont'd)

MEAN ATTITUDES OF MANAGER AND WORKERS RELATED TO THE COST-EFFECTIVENESS OF 28 UPPOE COMPANIES

MEAN ATTITUDES	Wage Cost Per Sales Income	Inventory Per Sales Income	Advertising Per Sales Inc	Extra Market Activity Per Sales Income	Materials Used P.Sales Income	Gross Return on Investment Plus Compens.	Total Expenses	Operating Profit
Workers (cont'd)								
Satis. with Company Operations	.16	-.56*	-.71*	-.09	-.25	.62*	.47*	.53*
Satis. with Boss	.23	-.26	-.37°	-.03	.06	.43°	.28	.34
Overlapping Responsibility	-.25	.34	.39°	.24	-.07	-.49*	-.54*	-.35
Satis. with Subordinates	.04	-.46°	-.46°	.11	-.11	.35	.31	.28
Clarity of Job Goals	-.09	-.58*	-.71*	-.16	-.22	.48*	.33	.57*
Opportunity for Own Decision	-.25	-.07	-.13	.16	-.05	.12	.13	.35
Extent of Dec Matching Resp.	-.29	.10	.07	.07	.03	.01	.13	.29
Upward Communication Failures	-.23	.18	.52*	-.01	.21	-.35	-.42°	-.27
Extent of Conflict of Interests	-.05	.17	.53*	-.13	.43°	-.25	-.22	-.29
Individual, not Group Decisions	-.06	.06	.47*	.11	-.59*	.11	.41°	.17

*N = 28, d.f. = 26, $p < .01$ when r = .47
°N = 28, d.f. = 26, $p < .05$ when r = .37

TABLE 4

MEAN PERSONNEL ATTRIB'''ES OF MANAGERS AND WORKERS
RELATED TO THE COST-EFFECTIVENESS OF 28 UPPOE COMPANIES

PERSONNEL ATTRIBUTES	Wage Cost Per Sales Income	Inventory Per Sales Income	Advertising Per Sales Inc	Extra Market Activity Per Sales Income	Materials Used P.Sales Inc.	Gross Return on Investment Plus Compens.	Total Expenses	Operating Profit
Managers								
Age	-.06	-.05	-.30	-.39°	.05	.28	.16	.35
Verbal Aptitude	.25	.02	-.17	.23	-.17	.34	.56°	.19
Quantitative Aptitude	.29	.13	-.00	.35	.05	-.05	.08	-.25
Verbal and Quantitative Aptitude	.35	.09	-.08	.27	-.09	.24	.51*	-.01
Self-Orientation	.20	.03	-.06	.29	-.18	.11	.32	.00
Interaction-Orientation	-.05	.18	.24	-.21	.16	-.05	-.14	-.14
Task-Orientation	-.17	-.26	-.08	.00	.07	-.21	-.30	-.02
Pro-Union Attitudes	-.12	-.18	.10	.07	-.16	.09	.38	.19
Amount of Work Experience	-.01	-.07	-.37°	-.33	.04	.33	.23	.35
Workers								
Age	.30	.19	-.04	-.37°	.24	.36	.31	.13
Verbal Aptitude	.27	.12	-.38°	-.18	-.27	.53*	.55°	.30
Quantitative Aptitude	.35	.19	-.17	.06	-.17	.40°	.54	.17
Verbal and Quantitative Aptitude	.41°	.19	-.35	.07	-.23	.48*	.56*	.20
Self-Orientation	.24	.19	-.17	.14	-.11	.15	.18	-.04
Interaction-Orientation	-.21	-.05	-.06	.02	-.17	-.13	-.23	-.01
Task-Orientation	.00	-.07	.15	-.08	.14	.05	.11	.06
Pro-Union Attitudes	.16	.15	-.43°	.25	-.64*	.47*	.49*	.29
Amount of Work Experience	.32	.19	-.19	-.32	.00	.38°	.26	.18

*N = 28, d.f. = 26, $p < .01$ when $r = .47$
°N = 28, d.f. = 26, $p < .05$ when $r = .37$

TABLE 5

INTERCORRELATIONS AMONG THE 10 FINANCIAL VARIABLES AND COMPANY POLICY

Variable	Sale Mkt	Sale Other	Raw Matl.	Sal. Wages	Equip.	Stor.	Off Purch.	Mkt. Res.	Adv.	Int.	"T-D" Policies
	1	2	3	4	5	6	7	8	9	10	11
Sales to Market		-.11	.83	.63	.01	.31	.07	.52	.82	.39	.21
Sales to Other Companies			-.12	-.08	-.17	-.15	-.09	-.15	-.11	-.19	.19
Raw Materials Used				.72	-.03	.27	.19	.43	.64	.36	.28
Salaries and Wages					-.03	.57	.42	.21	.56	.26	.31
Equipment Expenditures						.20	.23	.29	-.17	-.18	-.19
Storage Costs							.30	.09	.24	.40	.16
Off-Market Purchases								.11	.06	-.02	-.06
Market Research									.47	.51	.21
Advertising Costs										.38	.48
Interest Charges											.07
"Top-Down" Policies											

TABLE 6

ROTATED FACTOR MATRIX FOR
10 FINANCIAL VARIABLES

Variable	Gross Processing Activity F1	Variable Costs F2	Conservative Planning F3	Fixed Costs F4	Comm h^2
Sales to Market	.87	.21	-.12	.02	.81
Sales to Other Companies	-.02	-12	.73	-.15	.56
Raw Materials Used	.80	.33	-.04	-.01	.75
Salaries and Wages	.56	.74	.04	-10	.86
Equipment Expenditures	-.05	.13	-.08	.90	.85
Storage Costs	.19	.74	-.28	-.06	.67
Off-Market Purchases	-.02	.72	.05	.35	.65
Market Research	.72	-.18	-.27	.45	.82
Advertising Costs	.87	.13	.00	-.15	.80
Interest Charges	.49	.04	-.65	-.27	.73
"Top-Down" Policies	.54	.04	.48	.23	.57

TABLE 7

ROTATED FACTOR MATRIX FOR 14
MANAGER ATTITUDINAL VARIABLES

Variable	Satis. with Top-Down Mgmt & Policies	Satis. with Individual Resp. and Authority	Internally Responsive Company	Satis. with Climate & Operations	Comm.
	M1	M2	M3	M4	h^2
Job Satisfaction	.36	.12	-.23	.69	.68
Ease in Selling Ideas	.42	.29	.00	.75	.83
Ease of Openness	.32	.25	-.15	.71	.69
Satisfaction with Company Operations	-.16	.14	.16	.89	.87
Satisfaction with Boss	.72	-.12	.03	.29	.62
Perceived Overlapping Responsibilities	.44	-.03	-.51	-.58	.79
Satisfaction with Subordinates	-.03	.03	.22	.85	.78
Clarity of Job Goals	.46	.13	.09	.66	.67
Felt Opportunity for Own Decisions	-.10	.90	-.13	.21	.88
Felt Matching of Authority-Responsibility	-.20	.86	.13	.22	.85
Perceived Upward Communication Failures	-.09	-.54	-.17	-.60	.69
Felt Conflict of Interests	-.55	-.10	-.45	-.27	.58
Perceived Individual, Not Group Decisions	-.12	-.65	.49	.01	.68
Perceived Internally Responsive Company	.14	-.08	.85	.01	.75
Top-Down Policies	.85	-.15	.11	-.04	.76

TABLE 8

ROTATED FACTOR MATRIX FOR 14
WORKER ATTITUDINAL VARIABLES

Variable	Conflict Free Structure W1	Permissive Supervision W2	Top-Down Policies W3	Satis. with Working Conditions W4	Satis. with Resp. and Authority W5	Comm h^2
Job Satisfaction	-.10	.30	-.03	.66	.50	.79
Ease in Selling Ideas	.07	.83	.12	.33	.33	.93
Ease of Openness	.25	.79	-.01	.12	.37	.85
Satisfaction with Company Operations	.58	.34	-.18	.63	.09	.90
Satisfaction with Boss	.40	.42	-.17	.38	.15	.54
Perceived Overlapping Responsibilities	-.75	-.07	-.34	-.37	-.05	.82
Satisfaction with Subordinates	.03	.11	.12	.90	-.01	.84
Clarity of Job Goals	.47	.09	-.20	.65	.28	.78
Felt Opportunity for Own Decisions	.12	.36	.10	.21	.81	.86
Felt Matching of Authority-Responsibility	.13	.19	.06	.04	.93	.92
Perceived Upward Communication Failures	-.89	-.20	.10	-.03	-.06	.85
Felt Conflict of Interests	-.70	-.29	.14	.00	.03	.60
Perceived Individual, Not Group Decisions	-.30	-.28	.70	-.03	.17	.70
Perceived Internally Responsive Company	.76	-.29	.14	.15	.37	.85
Top-Down Policies	.17	.22	.90	.00	.01	.90

ABSTRACT

COMMITTEE DECISIONS*

Dr. Gordon Tullock

Committee activities enter into many Governmental decisions, including decisions related to cost/effectiveness. Although committees may be formed or activated for reasons other than reaching decisions, the following remarks are directed only (or in the main) to the latter class of cases. They are intended to summarize certain salient problems that have entered into research which the author now has under way in this area.

Recent work in application of economic models to political problems has dealt with collective decision procedures, but all of the existing models require that the group reach its decision by some formal voting procedure. They are not, therefore, directly applicable to the normal committee. One aspect of this theory, however, can be carried over: Individual members of decision groups aim at their own ends, which are not necessarily those given to the committee itself in the organization chart. Commonly in committees the individual member is primarily motivated by a desire to improve his status in the permanent organization which he represents in the committee. His role, and his behavior, is that of an ambassador attempting to get as much as possible for his principle. A second motivation may be a desire to sell something which the individual committee member values. Finally, the individual may, indeed, have internalized the ostensible objectives of the committee, and thus simply make decisions in terms of how well they fit that objective. This type of behavior I term judicial, and I think that it is rare without the very special precautions with which the constitution surrounds federal judges.

Committees can be classified largely by the type of motivation which dominates their members. Firstly, however, some organizations which look much like committees, but which really make no decisions must be eliminated. These organizations, and the conference of a learned society may be taken as the type case, exist for the exchange of information, not to plan action. Many ostensible committees actually are organizations to improve coordination by providing channels for information transfer rather than making decisions. Another committee-like organization is the entity which used to go under the name of "royal council." Any individual decision maker is likely to want advice and specialized counsel. An organization for giving such advice, the cabinet will do as an example, may easily be mistaken for a committee. The difference is that the president, not the cabinet, makes the decisions.

* This paper is not published in its entirety in this volume. Requests for further information should be addressed to the author, Dr. Gordon Tullock, Department of Economics, University of Virginia, Charlottesville, Va.

Most actual committees in the executive area are composed of people who act like ambassadors representing different organizations, and the committee itself functions much like a diplomatic conference. There is the same tendency to delay and seeking of the lowest common denominator. There are some intra-organization committees, however, and these do not, at least, have the strong conflicting interests which dominate the inter-organization committees. The appellate courts are the best examples of committees in which the members do not function as representatives of other organizations.

Probably the most predictable consequence of referring some decision to a committee is delay. It is also likely to lead to the eventual decision (if there is a decision) being a compromise. Sometimes the compromise will be merely verbal, a formula which permits different interpretations, but it may be genuine. If the people on the committee represent interests which are thought to have a legal or moral right to be conciliated, then the compromise may be the ideal outcome. This is, of course, the reason for the use of large committees, called legislatures, in most democratic governments.

The normal outcome of the use of committees to make administrative decisions is a reduction of individual responsibility for the decision, and a reduction in the quality of the decision making personnel. The last is a consequence of the fact that the money used to hire 5 committee members at $20,000 apiece could hire one man at $100,000 or a man at $40,000 with three advisors at $20,000. In either case (assuming that it is possible to judge ability) the actual decision maker would be of a higher level of capacity than anyone on the original committee. It is, I suppose, clear from my book, The Politics of Bureaucracy, that I think that the executive branch of the federal government is an appalling administrative mess. The widespread use of committees for decision making is both evidence and the result of this maladministration.

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